

3. $\Delta > 0$: $\Delta = 1 - 4r$

4. $\Delta = 0$: $\Delta = 1 - 4r = 0 \Rightarrow r = \frac{1}{4}$

5. $\Delta < 0$: $\Delta = 1 - 4r < 0 \Rightarrow r > \frac{1}{4}$

ا) $y = \frac{|x|+1}{|x|-1} \Rightarrow y|x| - |y| = |x|+1$

ب) $y = \frac{x^2+1}{|x|} \Rightarrow |x| + \frac{1}{|x|} = y$

$y|x| - |x| = 1 + |y| \Rightarrow |x|(y-1) = 1+|y|$

$\frac{1}{x} > 0 \Rightarrow 1 + \frac{1}{x} = y$

$|x| = \frac{1+|y|}{y-1} \Rightarrow \frac{1+|y|}{y-1} > 0 \Rightarrow \frac{-1}{+} - \frac{1}{-} +$

$R_f = [r, +\infty)$

$R_f = (-\infty, -\frac{1}{r}] \cup (1, +\infty)$

$y = \frac{x^2+bx-r}{x+r} \Rightarrow yx + ry = x^2 + bx - r$

$\Rightarrow x^2 + rx - yx - r - ry = 0 \Rightarrow x^2 + x(r-y) - (r+ry) = 0 \Rightarrow x = \frac{-b \pm \sqrt{\Delta}}{2a}$

$x = \frac{-(r-y) \pm \sqrt{(r-y)^2 - 4(-r-ry)}}{2} \Rightarrow (r-y)^2 - 4(-r-ry) > 0 \Rightarrow y^2 + 4 - 4y + 4r + 4ry > 0$

$y^2 - 4y + 4r > 0 \Rightarrow \Delta = 16 - 4(4r) < 0 \Rightarrow (I) + \infty$

$R_y = \mathbb{R}$

$f(x) = \frac{x^2-x+1}{x^2-x+1} \Rightarrow x^2-x+1 \neq 0 \Rightarrow \Delta = 1-4 < 0 \Rightarrow + \infty$

$D_f = \mathbb{R}$

$\frac{(x-\frac{1}{4})^2}{(x-\frac{1}{4})^2 + \frac{3}{4}} \Rightarrow \frac{t}{t+\frac{3}{4}} \Rightarrow + \infty \Rightarrow y > 0$
 $\downarrow$
 $\frac{0}{1} \Rightarrow y = 0$

$R_f = [0, 1)$

الف) $y = \sqrt{-x^2+4x-1}$

ب) $y = \sqrt[3]{x^2-1}$

$\left\{ \begin{array}{l} \frac{-b}{2a} \Rightarrow \frac{-4}{-2} = 2 \\ y_{max} = -1+4-1 = 2 \end{array} \right. \Rightarrow (2)$
 $R_y = \sqrt{[-\infty, 4]} \Rightarrow (0, \sqrt{4}]$

$\left\{ \begin{array}{l} \frac{-b}{2a} = \frac{-1}{1} = -1 \\ y_{min} = 1-1-1 = -1 \end{array} \right. \Rightarrow (-1)$

$R_y \Rightarrow \sqrt[3]{[-1, +\infty)} \Rightarrow [-1, +\infty)$

ا) $y = \log(x^2-x+1)$

ب) $y = \log_r\left(\frac{x^2+1}{x}\right) = y \Rightarrow yx = x^2+1$

$R_y = \log(\mathbb{R}) \Rightarrow R_y = (-\infty, +\infty)$

$x^2-yx+1 = 0 \Rightarrow \Delta = y^2 - 4 > 0 \Rightarrow y > 2$
 $y < -2, y < -2 \Rightarrow (-\infty, -2] \cup [2, +\infty)$

$R_y = \log_r(-\infty, -2] \cup [2, +\infty)$

$R_y = (-\infty,$

الف) $y = |x-1| - |x+1|$

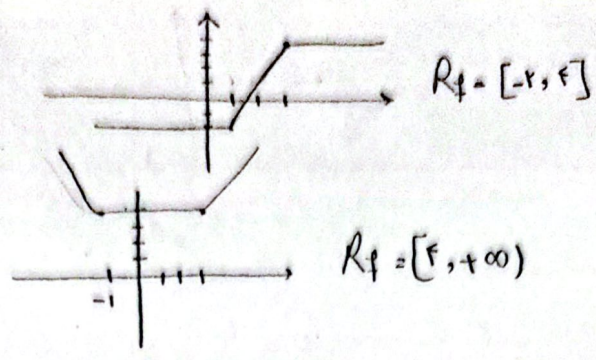
$x=1 \rightarrow y=0$

$x=-1 \rightarrow y=-2$

ب) $y = |x-1| + |x+1|$

$x=1 \rightarrow y=2$

$x=-1 \rightarrow y=0$



$|x-1| - |x+1| \leq 1$

$x < -1$	$-1 < x < 1$	$x > 1$
$1-x-1-x+1$	$1-x-1-x-1$	$x-1-1-x-1$
$-x-1$	$-2x-1$	-2
(I)	(II)	(III)

(I) $x < -1 \rightarrow x \leq -1 \rightarrow (-\infty, -1]$

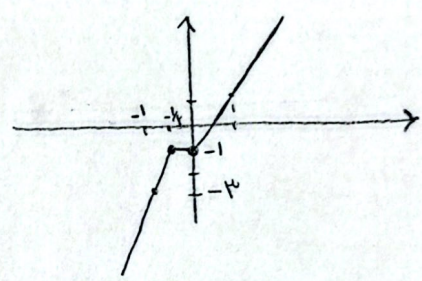
(II) $-1 < x < 1 \rightarrow x \geq -\frac{1}{2} \rightarrow [-\frac{1}{2}, 1]$

(III) $x > 1 \rightarrow x \leq 1 \rightarrow [1, +\infty)$
 $(-\infty, -1] \cup [-\frac{1}{2}, +\infty)$

$y = |x-1| + |x+1| + |x|$

$x < -1$	$-1 < x < 0$	$x > 0$
$1-x-1-x+1$	$1-x-1-x-1$	$x-1-x-1+x$
$-x$	$-2x-1$	$x-1$
(I)	(II)	(III)

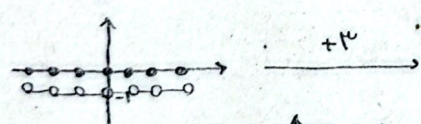
$x=-1 \rightarrow (-1)$



$R_f = \mathbb{R}$

الف) $y = [x] + [x-1] \rightarrow [x] + [-x] + 1$

$[x] + [-x] \in$



نلاحظ ان $[x] + [-x] \in [-1, 1]$ و $[x] + [-x] + 1 \in [0, 2]$



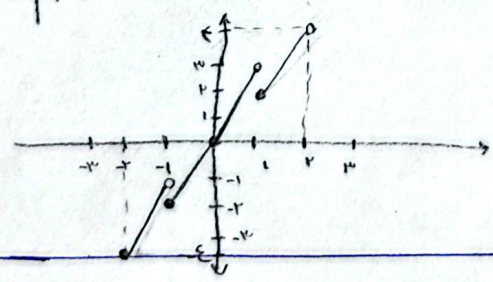
$[x] + [-x] = -1 \rightarrow [x] = -1$

ب) $y = [x] - [x]$

$[0, 1) \rightarrow [x] = 0$

$[1, 2) \rightarrow [x] = 1$

$[-1, 0) \rightarrow [x] = -1$



ج) $y = \sin^2 x - \sin x + 1$

$\sin x = -1 \rightarrow y = 1$ max

$\sin x = 1 \rightarrow y = 1$

$\sin x = \frac{-b}{2a} = \frac{1}{2} \rightarrow y = \frac{3}{4}$ min

$\Rightarrow R_y = [\frac{3}{4}, 1]$

د) $y = \sin^2 x + \sin x + 1$

$\sin^2 x = 0 \rightarrow y = 1$

$\sin^2 x = 1 \rightarrow y = 2$ max

$\sin^2 x = \frac{-b}{2a} = \frac{-1}{2} \rightarrow y = \frac{1}{2}$ min

$\Rightarrow R_y = [\frac{1}{2}, 2]$