

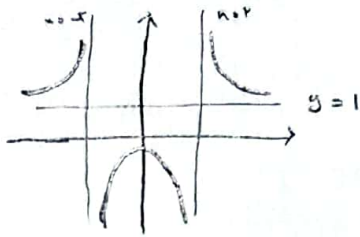
داریم رشتۀ B

شماره شصت و یک

سوره سدره

$$y = \frac{|x|+1}{|x|-2}$$

$$Ry = (-\infty, -\frac{1}{3}] \cup (1, +\infty) \quad \rightarrow y = \frac{x^2+1}{|x|} = \frac{|x|^2+1}{|x|}$$



$$|x| + \frac{1}{|x|} \Rightarrow Ry = [1, +\infty)$$

1

$$y = \frac{x^2+x-2}{x+2}$$

$$yx + xy = x^2 + x - 2$$

$$x^2 + (2-y)x + (-2-xy) = 0$$

$$(2-y)^2 - 4(-2-xy) \geq 0$$

$$4 + y^2 - 4y + 8 + 4xy \geq 0 \rightarrow y^2 + 4y + 12 \geq 0$$

$$y^2 + 4y + 12 = 0 \rightarrow y = -2 \quad y = -6$$

$$Ry = (-\infty, -6] \cup [2, +\infty)$$

2

$$f(x) = \frac{x^2-x+\frac{1}{2}}{x^2-x+1} = \frac{(x-\frac{1}{2})^2}{x^2-x+\frac{1}{2}+\frac{1}{2}} \rightarrow x - \frac{1}{2} = t \Rightarrow \frac{t}{t+\frac{1}{2}}$$

$$D_f = \mathbb{R} \quad \text{و} \quad \text{صورت}$$

$$yt + \frac{1}{2}y = t \rightarrow yt - t = -\frac{1}{2}y \rightarrow (y-1)t = -\frac{1}{2}y \rightarrow t = \frac{-\frac{1}{2}y}{y-1}$$

$$+ \text{و} \quad t \geq 0 \quad \frac{-\frac{1}{2}y}{y-1} \geq 0 \rightarrow \frac{0+1}{-\frac{1}{2}+\frac{1}{2}} \quad Ry = [0, +1)$$

3

$$\text{مثال } y = \sqrt{-x^2+4x+1} \quad \text{Max} \quad | \quad \frac{1}{2}$$

$$Ry = \sqrt{(-\infty, 1]}$$

$$Ry = [-1, \sqrt{2}]$$

$$\rightarrow y = \sqrt{x^2-4x+1} \quad \text{Min} \quad | \quad \frac{1}{2}$$

$$Ry = \sqrt{[1, +\infty)} \Rightarrow Ry = [-1, +\infty)$$

4

$$\text{مثال } y = \log(x^2-1+1)$$

$$Ry = \mathbb{R}$$

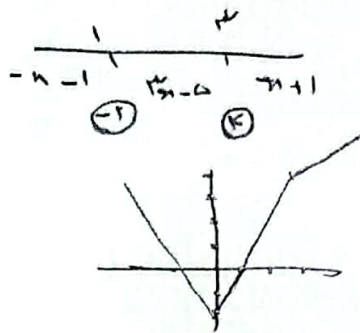
$$\rightarrow y = \log_r \frac{x^2+1}{x}$$

$$\frac{x^2+1}{x} = x + \frac{1}{x} \Rightarrow [1, +\infty)$$

$$Ry = [1, +\infty)$$

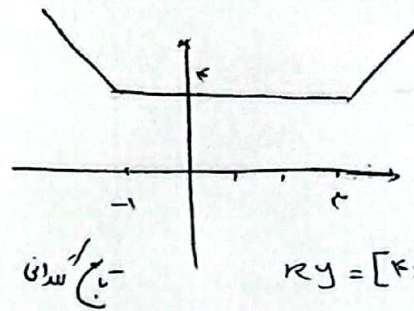
5

$$u) y = |x-1| - |x-2|$$



$$Ry = [-1, +\infty)$$

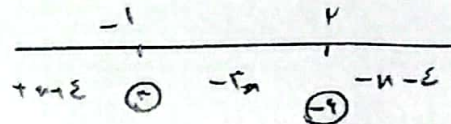
$$v) y = |x-1| + |x+1|$$



$$Ry = [2, +\infty)$$

6

$$|x-1| - |x+1| \leq 1$$



$$-x-1 \leq 1 \rightarrow x \geq -2 \rightarrow [-2, +\infty) \quad (1)$$

$$-x \leq 1 \rightarrow x \geq -1 \rightarrow [-1, +\infty) \quad (2)$$

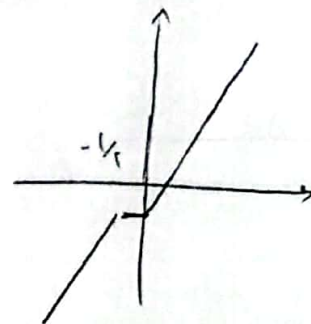
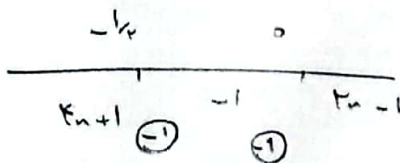
$$x+1 \leq 1 \rightarrow x \leq 0 \rightarrow (-\infty, 0] \quad (3)$$

$$1 \cap 2 \cap 3 \Rightarrow$$

$$(-\infty, 0] \cup [-1, +\infty)$$

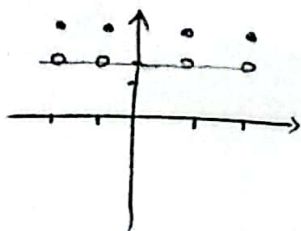
7

$$y = x - |x+1| + |x| \quad Ry = \mathbb{R}$$



8

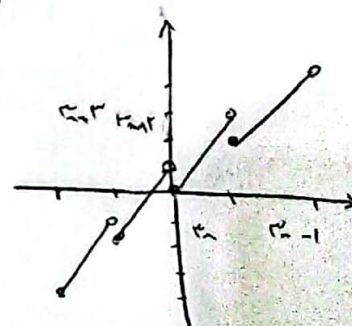
$$y = [x] + [x-n] = [x] + [-x] + n$$



$$Ry = \{r, r\}$$

$$v) y = x - [x]$$

$$Ry = [-1, 1)$$



9

$$u) y = \sin^2 x - \sin x + 1$$

$$1 \rightarrow 1$$

$$-1 \rightarrow r$$

$$-\frac{1}{4} \rightarrow \frac{1}{4} \rightarrow \frac{1}{8}$$

$$Ry = [\frac{1}{8}, 1]$$

$$v) y = \sin^2 x + \sin^2 x + 1$$

$$q \rightarrow 1$$

$$1 \rightarrow r$$

$$-\frac{1}{4} \rightarrow x$$

$$Ry = [1, 2]$$

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