

الف) $y = \frac{|x|+1}{|x|-2} \Rightarrow y|x|-2y = |x|+1 \Rightarrow y|x|-|x| = 2y+1 \Rightarrow$ (۲)

$|x|(y-1) = 2y+1 \Rightarrow |x| = \frac{2y+1}{y-1} \xrightarrow{x>0} + \frac{-1}{+1} + \frac{1}{-1} + \dots \Rightarrow R = (-\infty, \frac{1}{2}) \cup (1, +\infty)$

ب) $y = \frac{x^2+1}{|x|} \xrightarrow{x>0} y = x + \frac{1}{x} \xrightarrow{a+\frac{1}{x}} R = [2, +\infty)$

$y = \frac{x^2+2x-2}{x+3} \Rightarrow y(x+3) = x^2+2x-2 \Rightarrow x^2 + (2-y)x - (2y+2) = 0$ (۷) (۵.۵)

$\Delta \geq 0 \Rightarrow (2-y)^2 + 4(2y+2) = y^2 + 4y + 12 = (y+2)^2 + 8$

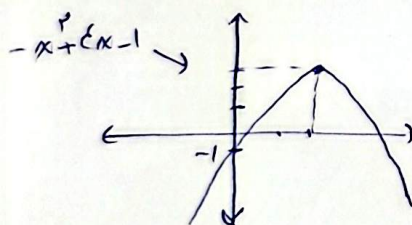
$(y+2)^2 + 8 > 0 \xrightarrow{\text{همیشه بزرگتر از ۰}} R_f = R \setminus (-\infty, -2) = \text{بر}$

صفحه آخر...

$f(x) = \frac{x^2-x+\frac{1}{x}}{x^2-x+1} \xrightarrow{\text{دانه}} \text{خرج از ۰} \neq \text{خرج} \Rightarrow x^2-x+1=0 \xrightarrow{\Delta < 0} \text{خرج از ۰ نیست} \Rightarrow D_f = R$ (۲)

$\xrightarrow{\text{بر}} \frac{(x-\frac{1}{2})^2}{(x-\frac{1}{2})^2 + \frac{3}{4}} = f(x) \xrightarrow{(x-\frac{1}{2})^2 \geq 0} f(x) \geq 0 \xrightarrow{\frac{f(x)}{1-f(x)} \text{ کسر همواره مثبت و کوچکتر از ۱}} R_f = [0, 1)$

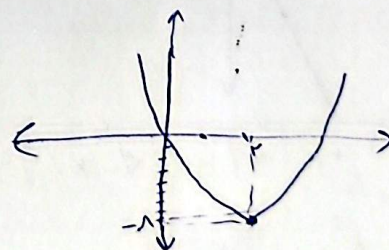
الف) $y = \sqrt{-x^2+4x-1} \xrightarrow{\text{رسم تابع}} -x^2+4x-1 \Rightarrow \frac{b}{2a} \rightarrow \frac{4}{-2} = -2 \quad \frac{-\Delta}{2a} \rightarrow \frac{-17}{-2} = 8.5$ (۲) (۱.۵) (۴)



$\xrightarrow{\text{برد تابع}} \sqrt{(-\infty, 3]} \Rightarrow R_f = [0, 3]$ (۳)

به روی خط بدنه ایمان می شود!

ب) $y = \sqrt[3]{2x^2-11x} \xrightarrow{\text{رسم تابع}} \frac{b}{2a} \rightarrow \frac{-11}{4} = -2.75 \quad \frac{-\Delta}{4a} \rightarrow \frac{-1}{4} = -0.25$ (۲) (۱)



$R_y = \sqrt[3]{[-1, +\infty)} \Rightarrow R_y = \sqrt[3]{[-1, +\infty)} \rightarrow R_y = [-1, +\infty)$

الف) $y = \lg(x^3-x+18) \xrightarrow{\text{برد تابع}} x^3-x+18 \text{ همواره مثبت} \Rightarrow R_f = (-\infty, +\infty)$ (۱) (۱)

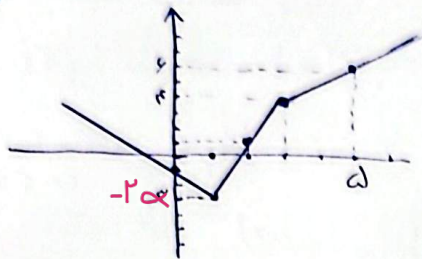
$\lg(-\infty, +\infty) \Rightarrow R_f = (-\infty, +\infty)$

صفحه آخر...

ب) $y = \lg\left(\frac{x^2+1}{x}\right) \Rightarrow y = \lg\left(x + \frac{1}{x}\right) \xrightarrow{a+\frac{1}{x}} \text{چون باید } 0 < x + \frac{1}{x} \text{ باشد به خاطر } \lg \text{ پس } R_f = [2, +\infty)$

بعد باید از ۰ بزرگتر بشود

$$c) y = |2x-1| - |x-2|$$



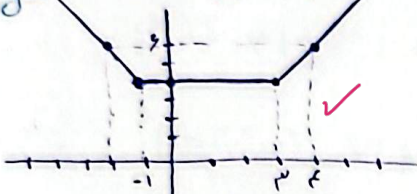
$$| -x-1 | \quad | x-2 | \quad | x+1 |$$

$$R_f = [-2, +\infty)$$

وقت!

(6) (15)

$$b) y = |x-1| + |x+1|$$



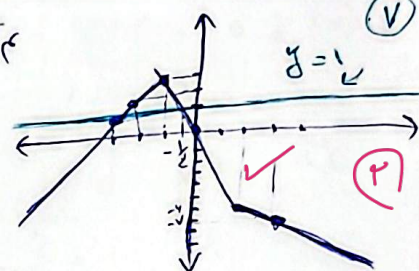
$$| -x+2 | \quad | x-2 |$$

$$R_f = [2, +\infty)$$

$$|x-1| - |2x+2| \leq 1 \xrightarrow{\text{ع.ع.م.}} |x+1| - 2|x-1|$$

$$\text{محدوده } \Rightarrow (-\infty, -1] \cup [-\frac{1}{2}, +\infty)$$

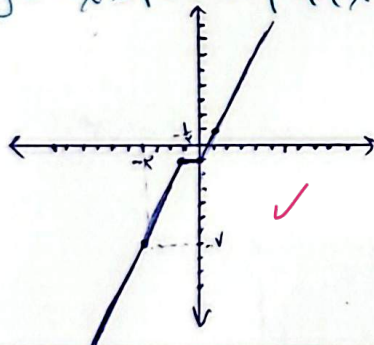
طبق نمودار



$$-2x = 1$$

$$x = -\frac{1}{2}$$

$$y = 2x - |2x+1| + |x|$$



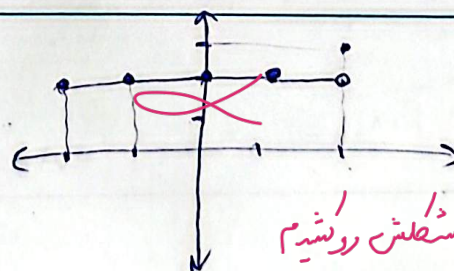
$$\xrightarrow{\text{ع.ع.م.}} |x+1| - 1 | 2x-1 |$$

$$\xrightarrow{\text{بردار}} (-\infty, +\infty)$$

(2) (1)

$$a) y = \{x\} + \{2-x\} \Rightarrow \begin{cases} x \in (-2, -1) \Rightarrow 2 \\ x \in [-1, 0) \Rightarrow 1 \\ x \in [0, 1) \Rightarrow 1 \\ x \in [1, 2) \Rightarrow 2 \\ x \in \{2\} \Rightarrow 2 \end{cases}$$

$$R_f = \{1, 2\}$$

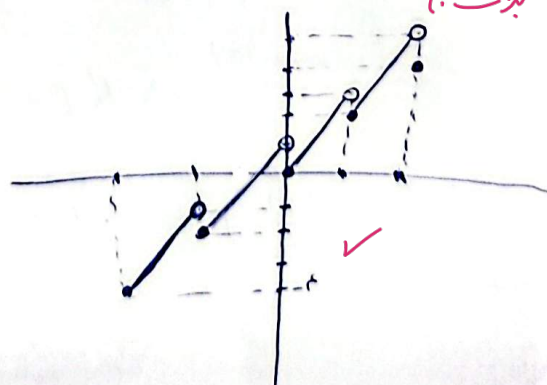


(2) (1)

صفحه آخر شطرنج رو کشیدم
برای:

$$b) y = 2x - \{x\} \Rightarrow \begin{cases} x \in [-2, -1) \Rightarrow 2x+2 \Rightarrow \\ x \in [-1, 0) \Rightarrow 2x+1 \Rightarrow \\ x \in [0, 1) \Rightarrow 2x \Rightarrow \\ x \in [1, 2) \Rightarrow 2x-1 \Rightarrow \\ x \in \{2\} \Rightarrow 2x-2 \Rightarrow \end{cases}$$

$$R_f = [-2, 2]$$



الف) $\sin^2 x - \sin x + 1$

$$\left. \begin{array}{l} \sin x \rightarrow 1 \Rightarrow 1 - 1 + 1 = 1 \\ \sin x \rightarrow -1 \Rightarrow 1 + 1 + 1 = 3 \\ -\frac{b}{2a} \rightarrow \frac{1}{2} \Rightarrow \frac{1}{4} - \frac{1}{2} + 1 = \frac{3}{4} \end{array} \right\}$$

$$\frac{3}{4} \leq \sin x \leq 1 \Rightarrow R_f = \left[\frac{3}{4}, 1 \right]$$

(1,5)

ب) $\sin^4 x + \sin^2 x + 1$

$$\left. \begin{array}{l} \sin^2 x \rightarrow 1 \Rightarrow 1 + 1 + 1 = 3 \\ \sin^2 x \rightarrow -1 \Rightarrow 1 + 1 + 1 = 3 \\ -\frac{b}{2a} \rightarrow 0 \Rightarrow 0 + 0 + 1 = 1 \\ \quad \quad \quad -\frac{1}{2} \Rightarrow 0 + 0 + 1 = 1 \end{array} \right\}$$

این مقدار یکتا!!

$$1 \leq \sin^2 x \leq 3 \quad ??$$

$$R_f = [1, 3]$$

$$1 \leq \sin x \leq 1 \Rightarrow R_f$$

صفحه آخر...

طرفین و ضمیمه

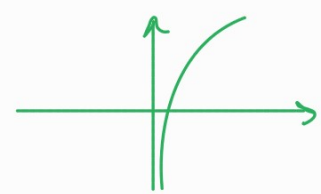
$$y^2 + 12y = x^2 + 12x - 2 \rightarrow x^2 + (12 - y)x - 2 - 12y = 0$$

$$x = \frac{y - 12 \pm \sqrt{(12 - y)^2 - 4(-2 - 12y)}}{2} = \frac{y - 12 \pm \sqrt{y^2 + 18y + 12}}{2}$$

$$\Delta \geq 0 \rightarrow y^2 + 18y + 12 \geq 0 \rightarrow \frac{-9}{+} \frac{-2}{-} \quad R_y = (-\infty, -4] \cup [-2, +\infty)$$

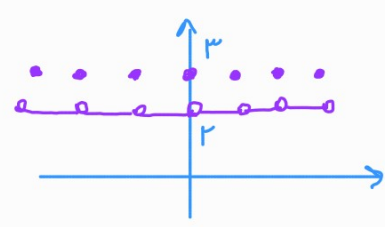
الف) برد توابع چندجمله‌ای از مرتبه فرد برابر $\mathbb{R}$ می‌باشد:

$$R_y = \text{Im}(-\infty, +\infty) = \text{Im}(0, +\infty) = (-\infty, +\infty) = \mathbb{R}$$



ب) $\frac{x^2 + 1}{x} = x + \frac{1}{x} \quad \begin{matrix} a + \frac{1}{a} \geq 2 \\ a + \frac{1}{a} \leq -2 \end{matrix} \rightarrow R = (-\infty, -2] \cup [2, +\infty)$
 چون متقارن بود پس $\log$ نیست اصلاً!
 $\xrightarrow{\log_2} R_y = [1, +\infty)$

الف) $y = [x] + [-x] + 2$



$$R_y = \{2, 3\}$$

$$y = \sin^2 x - \boxed{\sin x} + 1$$

$$\sin x = -1 \rightarrow y = 3$$

$$\sin x = 1 \rightarrow y = 1$$

$$\sin x = \frac{-b}{\pm a} = \frac{1}{2} \rightarrow y = \frac{3}{4}$$

$$\left. \begin{matrix} \sin x = -1 \rightarrow y = 3 \\ \sin x = 1 \rightarrow y = 1 \\ \sin x = \frac{1}{2} \rightarrow y = \frac{3}{4} \end{matrix} \right\} R_y = [\frac{3}{4}, 3]$$

$$y = \sin^r n + \boxed{\sin^r n} + 1$$

ب.ا.

$$\left. \begin{array}{l} \sin^r n = 0 \rightarrow y = 1 \\ \sin^r n = 1 \rightarrow y = 3 \\ \sin^r n = -\frac{b}{r_a} = -\frac{1}{r} x \end{array} \right\} R_y = [1, 3]$$