

الف) $y = \frac{|x|+1}{|x|-2} \Rightarrow y|x|-2y = |x|+1 \Rightarrow y|x|-|x| = 2y+1 \Rightarrow$ ①

$|x|(y-1) = 2y+1 \Rightarrow |x| = \frac{2y+1}{y-1} \xrightarrow{x>0} + \frac{-1}{+1-y} + \Rightarrow R = (-\infty, \frac{1}{2}] \cup (1, +\infty)$

ب) $y = \frac{x^2+1}{|x|} \xrightarrow{x>0} y = x + \frac{1}{x} \xrightarrow{a+\frac{1}{x}} R = [2, +\infty)$

$y = \frac{x^2+2x-2}{x+3} \Rightarrow y(x+3) = x^2+2x-2 \Rightarrow x^2 + (3-y)x - (2y+2) = 0$ ②

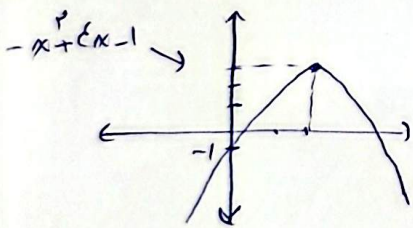
$\Delta \geq 0 \Rightarrow b^2 - 4ac \Rightarrow (3-y)^2 + 4(2y+2) = y^2 + 4y + 17 = (y+2)^2 + 9 > 0$

$(y+2)^2 + 9 > 0 \xrightarrow{\text{همیشه مثبت}} R_f = R = (-\infty, +\infty) = R$

$f(x) = \frac{x^2-x+\frac{1}{x}}{x^2-x+1} \xrightarrow{\text{دانه}} \text{خرج از مش} = x^2-x+1=0 \xrightarrow{\Delta < 0} D_f = R \downarrow$ ③

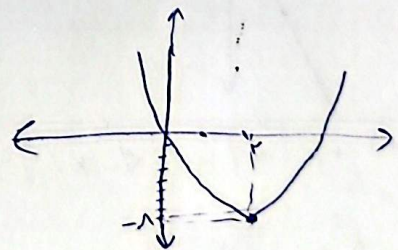
$\xrightarrow{R} \frac{(x-\frac{1}{2})^2}{(x-\frac{1}{2})^2 + \frac{3}{4}} = f(x) \xrightarrow{(x-\frac{1}{2})^2 \geq 0} f(x) \geq 0 \xrightarrow{\frac{f(x) < 1}{\text{کسر صحیح}}} R_f = [0, 1)$

الف) $y = \sqrt{-x^2+4x-1} \xrightarrow{\text{تابع}} -x^2+4x-1 \Rightarrow \frac{b}{2a} \rightarrow \frac{-4}{-2} = 2$ ④



$\xrightarrow{\text{بردار}} \sqrt{(-\infty, 3]} \Rightarrow R_f = [0, 3]$

ب) $y = \sqrt[3]{2x^2-11x} \xrightarrow{\text{تابع}} \frac{-b}{2a} \rightarrow \frac{-11}{4} = -\frac{11}{4}$ ⑤



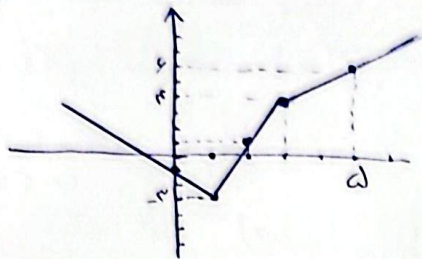
$R_y = \sqrt[3]{[-11, +\infty)} \Rightarrow R_y = [-11, +\infty)$

الف) $y = \lg(x^3-x+11) \xrightarrow{\text{برد تابع}} R \xrightarrow{\text{چون تابع درجه ۳}} R_f = (-\infty, +\infty) = R$ ⑥

$\lg(-\infty, +\infty) \Rightarrow R_f = [0, +\infty)$

ب) $y = \lg\left(\frac{x^2+1}{x}\right) \Rightarrow y = \lg\left(x + \frac{1}{x}\right) \xrightarrow{a+\frac{1}{x}} \text{چون } a+\frac{1}{x} > 0 \text{ باشد به شرط } R_f = [2, +\infty) \leftarrow \text{پس}$

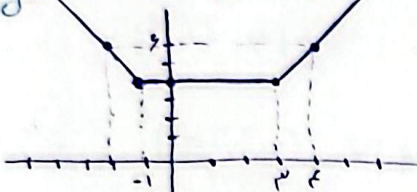
$$c) y = |x-1| - |x-2|$$



$$\begin{array}{c|c|c} -x-1 & x-2 & x+1 \\ \hline R_f = [-2, +\infty) \end{array}$$

(4)

$$b) y = |x-1| + |x+1|$$

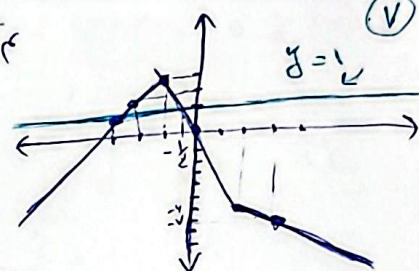


$$\begin{array}{c|c|c} -x+1 & x & x-1 \\ \hline R_f = [1, +\infty) \end{array}$$

$$|x-1| - |x+1| \leq 1 \xrightarrow{\text{ع.ع.م}}, \begin{array}{c|c|c} -x+1 & -x & -x-1 \\ \hline \end{array}$$

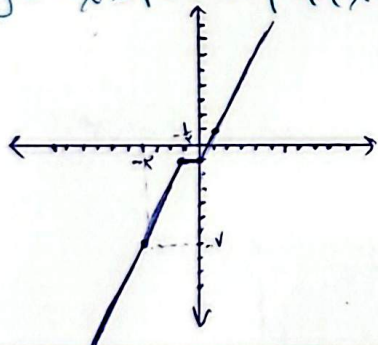
$$\text{ب.ع.م} \Rightarrow (-\infty, -1] \cup [-\frac{1}{2}, +\infty)$$

طبق



$$\begin{array}{l} -x = 1 \\ x = -1 \end{array}$$

$$y = x - |x+1| + |x|$$

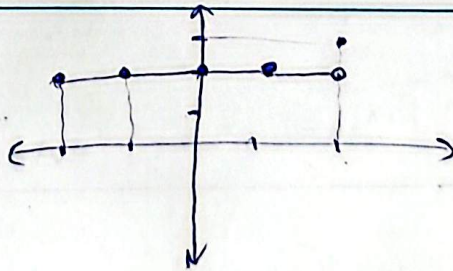


$$\xrightarrow{\text{ع.ع.م}}, \begin{array}{c|c|c} -x & -1 & x-1 \\ \hline \end{array}$$

$$\xrightarrow{\text{ب.ع.م}} (-\infty, +\infty)$$

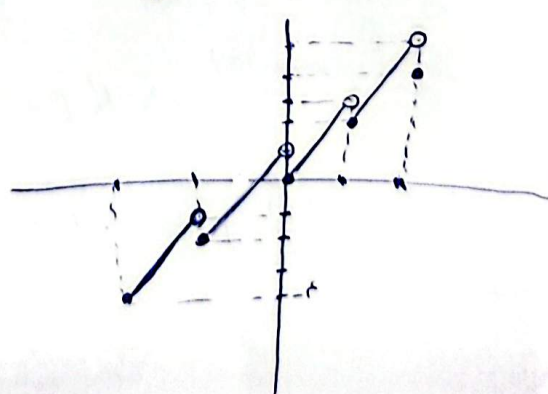
$$a) y = \{x\} + \{1-x\} \Rightarrow \begin{cases} x \in (-1, -1) = 2 \\ x \in [-1, 0) = 1 \\ x \in [0, 1) = 1 \\ x \in [1, 1) = 2 \\ x \in \{1\} = 0 \end{cases}$$

$$R_f = \{1, 2\}$$



$$b) y = x - \{x\} \Rightarrow \begin{cases} x \in (-1, -1) = x+1 \\ x \in [-1, 0) = x+1 \\ x \in [0, 1) = x \\ x \in [1, 1) = x-1 \\ x \in \{1\} = x-1 \end{cases}$$

$$R_f = [-1, 1)$$



الف) $\sin^2 x - \sin x + 1$

$$\left. \begin{array}{l} \sin x \rightarrow 1 \Rightarrow 1 - 1 + 1 = \boxed{1} \\ \sin x \rightarrow -1 \Rightarrow 1 + 1 + 1 = \boxed{3} \\ -\frac{b}{2a} \rightarrow \frac{1}{2} \Rightarrow \frac{1}{4} - \frac{1}{2} + 1 = \boxed{\frac{3}{4}} \end{array} \right\} \frac{\sqrt{3}}{2} \leq \sin x \leq \sqrt{3} \Rightarrow R_f = \left[\frac{\sqrt{3}}{2}, \sqrt{3} \right]$$

ب) $\sin^4 x + \sin^2 x + 1$

$$\left. \begin{array}{l} \sin^2 x \rightarrow 1 \Rightarrow 1 + 1 + 1 = \boxed{3} \\ \sin^2 x \rightarrow -1 \Rightarrow \text{invalid} \\ -\frac{b}{2a} \rightarrow 0 \Rightarrow 0 + 0 + 1 = \boxed{1} \\ \sin^2 x \rightarrow -\frac{1}{2} \Rightarrow \text{invalid} \end{array} \right\} \begin{array}{l} 1 \leq \sin^2 x \leq 3 \\ 1 \leq \sin x \leq \sqrt{3} \Rightarrow R_f \end{array}$$