

۱) $y = \frac{|x|+1}{|x|-2} \rightarrow \begin{cases} \text{مقدار مخرج} = 0 \rightarrow y = -\frac{1}{2} \\ \text{مقدار مخرج} = +\infty \rightarrow y = 1 \end{cases} \Rightarrow R_y = (-\infty, -\frac{1}{2}) \cup (1, +\infty)$

۲) $y = \frac{x^2+1}{|x|} \rightarrow y = |x| + \frac{1}{|x|} \rightarrow |x| + \frac{1}{|x|} \geq 2 \Rightarrow R_y = [2, +\infty)$
 $a > 0 \rightarrow a + \frac{1}{a} \geq 2$

۳) $y = \frac{x^2+2x-2}{x+3} \rightarrow yx+3y = x^2+2x-2 \rightarrow x^2+(2-y)x-(2+3y) = 0$
 $x = \frac{-(2-y) \pm \sqrt{(2-y)^2 + 4(2+3y)}}{2} \geq 0 \rightarrow (2-y)^2 + 4(2+3y) \geq 0 \rightarrow y^2 - 4y + 4 + 8 + 12y \geq 0 \Rightarrow$
 $y^2 + 8y + 12 \geq 0 \rightarrow (y+2)(y+6) \geq 0 \Rightarrow \begin{cases} y \leq -6 \\ y \geq -2 \end{cases} \Rightarrow R_y = \mathbb{R} - (-6, -2)$

۴) $f(x) = \frac{x^2-x+\frac{1}{4}}{x^2-x+1} \quad x^2-x+1 \neq 0 \rightarrow \Delta = 1-4(1)(1) = -3 < 0 \rightarrow$ مخرج نمی تواند صفر شود پس محدوده ی در آنده نداریم $\Rightarrow D_f = \mathbb{R}$
 $\hookrightarrow f(x) = \frac{(x-\frac{1}{2})^2}{(x-\frac{1}{2})^2 + \frac{3}{4}} \rightarrow f(x) = \frac{u^2}{u^2 + \frac{3}{4}} \rightarrow \begin{cases} \text{مقدار مخرج} = 0 \rightarrow y = 0 \\ \text{مقدار مخرج} = +\infty \rightarrow y = 1 \end{cases}$
 $\Rightarrow R_f = [0, 1)$

۵) $y = \sqrt{-x^2+4x-1} \rightarrow y_{\max} = \frac{-\Delta}{4a} = \frac{-(14-4(-1)(-1))}{-4} = 3 \Rightarrow R_y = \sqrt{[-\infty, 3]} \rightarrow R_y = [0, \sqrt{3}]$

۶) $y = \sqrt[3]{2x^2-8x} \rightarrow y_{\min} = \frac{-\Delta}{4a} = \frac{-(48-4(2)(0))}{4} = -12 \Rightarrow R_y = \sqrt[3]{[-\infty, 0]} \rightarrow R_y = [-12, +\infty)$

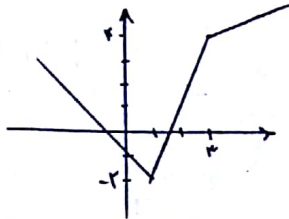
۷) $y = \log(\underbrace{x^2-x+18}_{\text{خیزد به بالا و صفر نمی زند}}) \rightarrow R_y = \log(-\infty, +\infty) = \mathbb{R}$

۸) $y = \log_r(\frac{x^2+1}{x}) \quad \frac{x^2+1}{x} = x + \frac{1}{x} \rightarrow \begin{cases} x + \frac{1}{x} \geq 2 \\ x + \frac{1}{x} \leq -2 \end{cases} \Rightarrow R = (-\infty, -2] \cup [2, +\infty)$
 $R_y = [1, +\infty) \xleftarrow{\log_r}$

$$u) y = |x-1| - |x-2|$$

$$\begin{cases} x+1 & x > 2 \\ x-2 & 1 \leq x \leq 2 \\ -x-1 & x < 1 \end{cases}$$

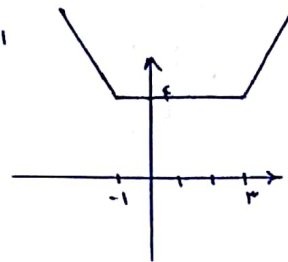
$$R_y = [-1, +\infty)$$



$$v) y = |x-2| + |x+1|$$

$$\begin{cases} x+2 & x > 2 \\ x-1 & -1 \leq x \leq 2 \\ -x+1 & x < -1 \end{cases}$$

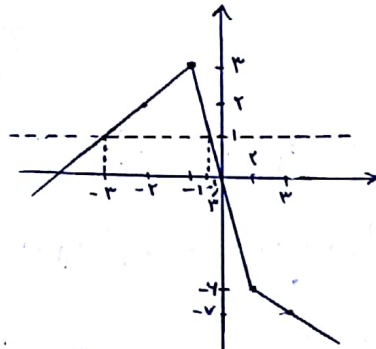
$$R_y = [1, +\infty)$$



6

$$|x-2| - |x+2| \leq 1$$

$$\begin{cases} -x-2 & x > 2 \\ -x & -1 \leq x \leq 2 \\ x+2 & x < -1 \end{cases}$$

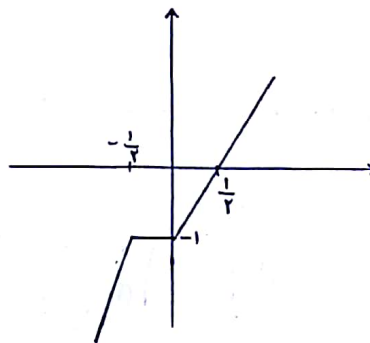


$$(-\infty, -2] \cup [-\frac{1}{2}, +\infty)$$

7

$$y = 2x - |x+1| + |x|$$

$$\begin{cases} x-1 & x > 0 \\ -1 & -\frac{1}{2} \leq x \leq 0 \\ 2x+1 & x < -\frac{1}{2} \end{cases}$$



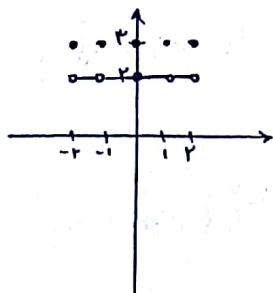
$$R_f = \mathbb{R}$$

8

$$u) y = [x] + [2-x] \rightarrow y = [x] + [-x] + 2$$

$$\begin{cases} 2 & x \in \mathbb{Z} \\ 1 & x \notin \mathbb{Z} \end{cases}$$

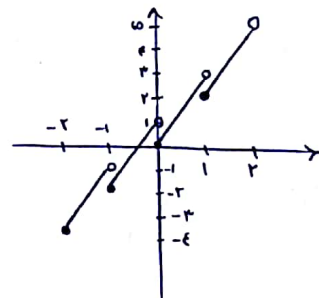
$$R_y = \{1, 2\}$$



$$v) y = 2x - [x]$$

$$\begin{cases} x-2 & x=2 \\ x-1 & 1 \leq x < 2 \\ x & 0 \leq x < 1 \\ x+1 & -1 \leq x < 0 \\ x+2 & -2 \leq x < -1 \end{cases}$$

$$R_y = [-2, \infty)$$



9

$$u) y = \sin^2 x - \sin x + 1 \rightarrow \square = \sin x \rightarrow \begin{cases} -1 \\ 1 \\ \frac{1}{2} \end{cases}$$

$$R_y = [\frac{1}{4}, 2]$$

$$v) y = \sin^2 x + \sin^2 x + 1 \rightarrow \square = \sin^2 x \rightarrow \begin{cases} 1 \\ 0 \\ \frac{1}{4} \end{cases}$$

$$R_y = [1, 2]$$

10