

نام و نام خانوادگی: پاسخنامه تشریحی تکلیف شماره ۸... کلاس: رتبه: رشته: B

الف) $y = \ln x + 1 \Rightarrow \ln x = y - 1$
 $\frac{1}{x} = \frac{1}{e^{y-1}} \Rightarrow R_f = (-\infty, -\frac{1}{e}] \cup (\frac{1}{e}, +\infty)$

ب) $y = \frac{x^2+1}{\ln x} = \ln x + \frac{1}{\ln x}$
 $a + \frac{1}{a} \geq 2 \Rightarrow R_f = [2, +\infty)$

$y = \frac{x^2+x-2}{x+2} \Rightarrow yx + 2y = x^2+x-2 \Rightarrow x^2 + (1-y)x - 2y - 2 = 0$
 $\Delta = (1-y)^2 - 4(-2y-2) = 1 - 2y + y^2 + 8y + 8 = y^2 + 6y + 9 = (y+3)^2 \geq 0$
 $\Rightarrow (y+3)(y+1) \geq 0 \Rightarrow R_f = (-\infty, -3] \cup [-1, +\infty)$

$f(x) = \frac{x^2-x+\frac{1}{x}}{x^2-x+1} \rightarrow \Delta = 1-4+\frac{4}{x} < 0 \rightarrow Df = \mathbb{R}$
 $yx^2 - yx + y = x^2 - x + \frac{1}{x} \Rightarrow (y-1)x^2 + (-y+1)x + y - \frac{1}{x} = 0$
 $\frac{y-1}{y} \rightarrow x = \frac{y-1 \pm \sqrt{(y-1)^2 - 4(y-1)(y-\frac{1}{y})}}{2(y-1)}$
 $\Rightarrow (y-1)^2 - 4(y-1)(y-\frac{1}{y}) \geq 0 \Rightarrow y^2 - y + 1 - 4y^2 + 4y - \frac{4}{y} = -3y^2 + 3y - \frac{4}{y} = -3y(y-1) - \frac{4}{y}$
 $\Rightarrow R = [0, 1]$

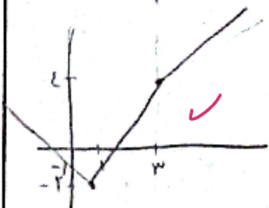
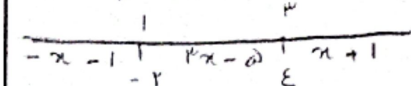
الف) $y = \sqrt{-x^2 + 8x - 1}$
 $y = -x^2 + 8x - 1 \rightarrow \max \begin{cases} -\frac{b}{2a} = -\frac{8}{-2} = 4 \\ -\frac{b^2}{4a} = -\frac{64}{-4} = 16 \end{cases} \Rightarrow R_f = [0, 4]$

ب) $y = \sqrt{2x^2 - 8x}$
 $y = 2x^2 - 8x \rightarrow \min \begin{cases} -\frac{b}{2a} = -\frac{-8}{4} = 2 \\ \frac{4ac - b^2}{4a} = \frac{0 - 64}{4} = -16 \end{cases} \Rightarrow R_f = [-16, +\infty)$

الف) $y = \log(x^2 - x + 1)$
 $x^2 - x + 1 \in \mathbb{R} \Rightarrow R = \mathbb{R}$
 $\Rightarrow R_f = \mathbb{R}$

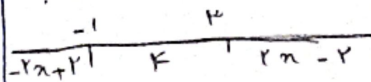
ب) $y = \log \left(\frac{x^2+1}{x} \right)$
 $\frac{x^2+1}{x} = x + \frac{1}{x} \geq 2 \Rightarrow R_f = [1, +\infty)$

الف) $y = |x_n - x| - |x - 3|$



$R_f = [-2, +\infty)$

ب) $y = |x - 3| + |x + 1|$

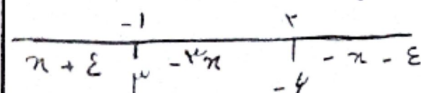


$R_f = [2, +\infty)$

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$|x - 3| - |x + 1| \leq 1$



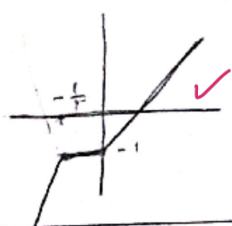
$\Rightarrow x \geq -\frac{1}{2}$
 $x \leq -3$

$R_f = (-\infty, -3] \cup [-\frac{1}{2}, +\infty)$

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$y = 3x - |x + 1| + |x|$

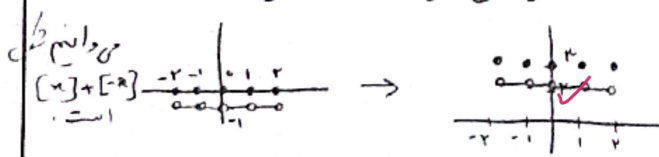


$R_f = \mathbb{R}$

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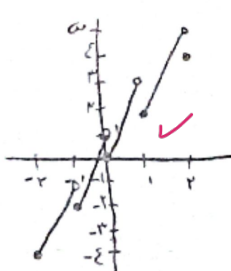
الف) $[x] + [3 - x] = 3 + [x] + [-x]$



$R = \{2, 3\}$

ب) $3x - [x]$

$-1 \leq x < -\frac{1}{2} \rightarrow 3x + 2$
 $[-x] = 2$
 $-\frac{1}{2} \leq x < 0 \rightarrow 3x + 1$
 $[-x] = 1$
 $0 \leq x < 1 \rightarrow 3x$
 $[-x] = 0$
 $1 \leq x < 2 \rightarrow 3x - 1$
 $[-x] = -1$



$R_f = [-2, 2)$

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الف) $y = \sin^2 x - \sin x + 1$

$\min \sin x = -1 \rightarrow 1 + 1 + 1 = 3$

$\max \sin x = 1 \rightarrow 1 - 1 + 1 = 1$

$-\frac{b}{2a} = -\frac{1}{2} = \frac{1}{2} \rightarrow \frac{1}{4} - \frac{1}{4} + 1 = \frac{3}{2}$

$R_f = [\frac{3}{2}, 3]$

ب) $\sin^2 x + \sin^2 x + 1$

$\min \sin^2 x = 0 \rightarrow 0 + 0 + 1 = 1$

$\max \sin^2 x = 1 \rightarrow 1 + 1 + 1 = 3$

$-\frac{b}{2a} = -\frac{1}{2} \rightarrow \frac{1}{4}$

$R_f = [1, 3]$

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