

$$\text{الف) } f(n) = r \sin^r \left(n + \frac{\pi}{r} \right) - 1 \quad -1 \leq \sin \left(n + \frac{\pi}{r} \right) \leq 1 \quad (1)$$

$$0 \leq \sin^r \left(n + \frac{\pi}{r} \right) \leq 1 \quad 0 \leq r \sin^r \left(n + \frac{\pi}{r} \right) \leq r \rightarrow -1 \leq r \sin^r \left(n + \frac{\pi}{r} \right) - 1 \leq 1$$

$$R_f = [-1, 1]$$

$$\rightarrow g(n) = r \cos^r n - r \quad -1 \leq \cos n \leq 1 \quad -1 \leq \cos^r n \leq 1$$

$$-r \leq r \cos^r n \leq r \quad -r \leq r \cos^r n - r \leq -1$$

$$R_g = [-r, -1]$$

$$y = \frac{n}{|n|} + [n]$$

$$n = r \rightarrow [n] = r \rightarrow y = 1 + r = r \quad (2)$$

$$-r \leq n < -r \quad [n] = -r \rightarrow y = -1 - r = -r$$

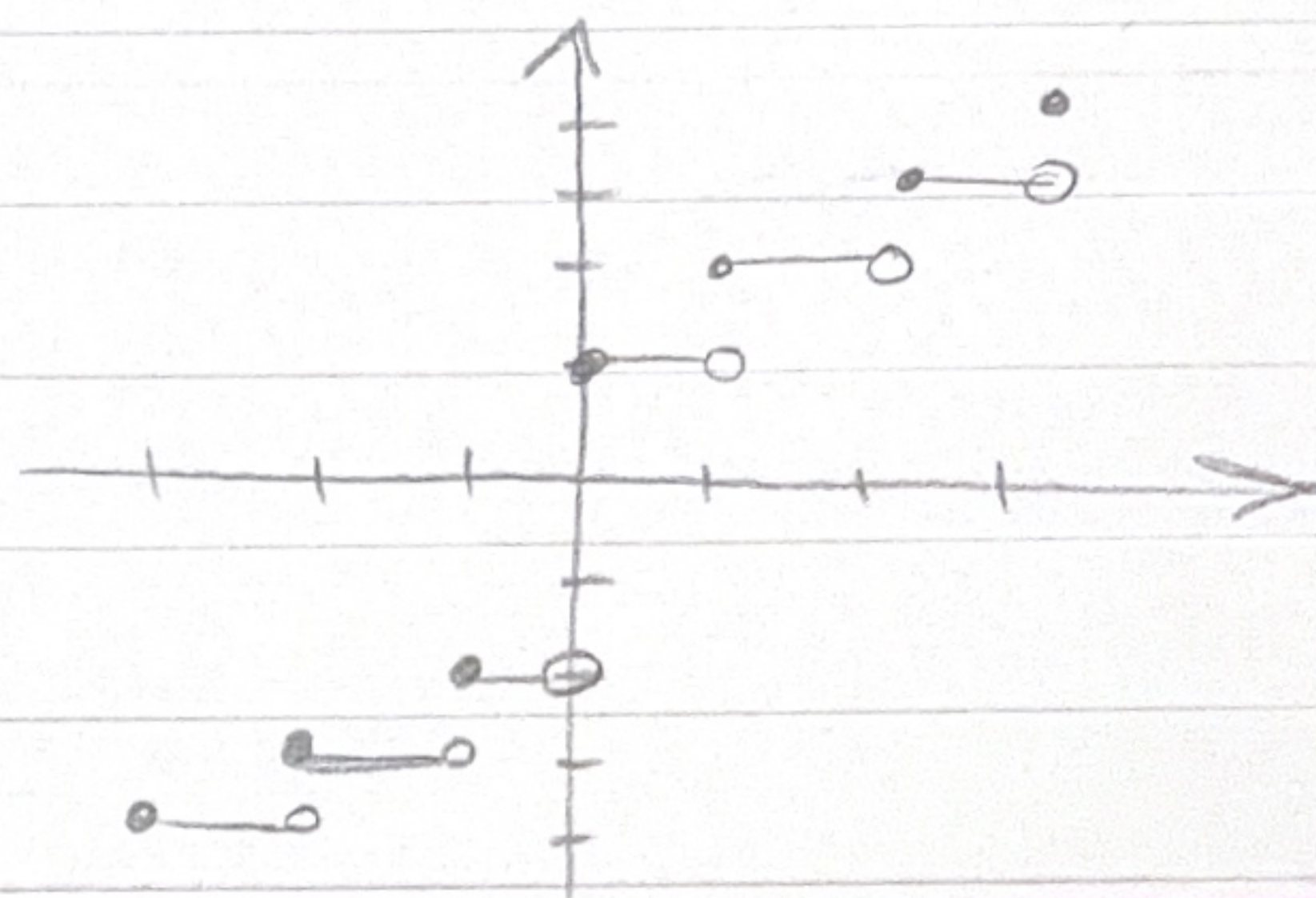
$$-r \leq n < -1 \quad [n] = -r \rightarrow y = -1 - r = -r$$

$$-1 \leq n < 0 \quad [n] = -1 \rightarrow y = -1 - 1 = -r$$

$$0 \leq n < 1 \quad [n] = 0 \rightarrow y = 1$$

$$1 \leq n < r \quad [n] = 1 \rightarrow y = 1 + 1 = r$$

$$r \leq n < r \quad [n] = r \rightarrow y = 1 + r = r$$

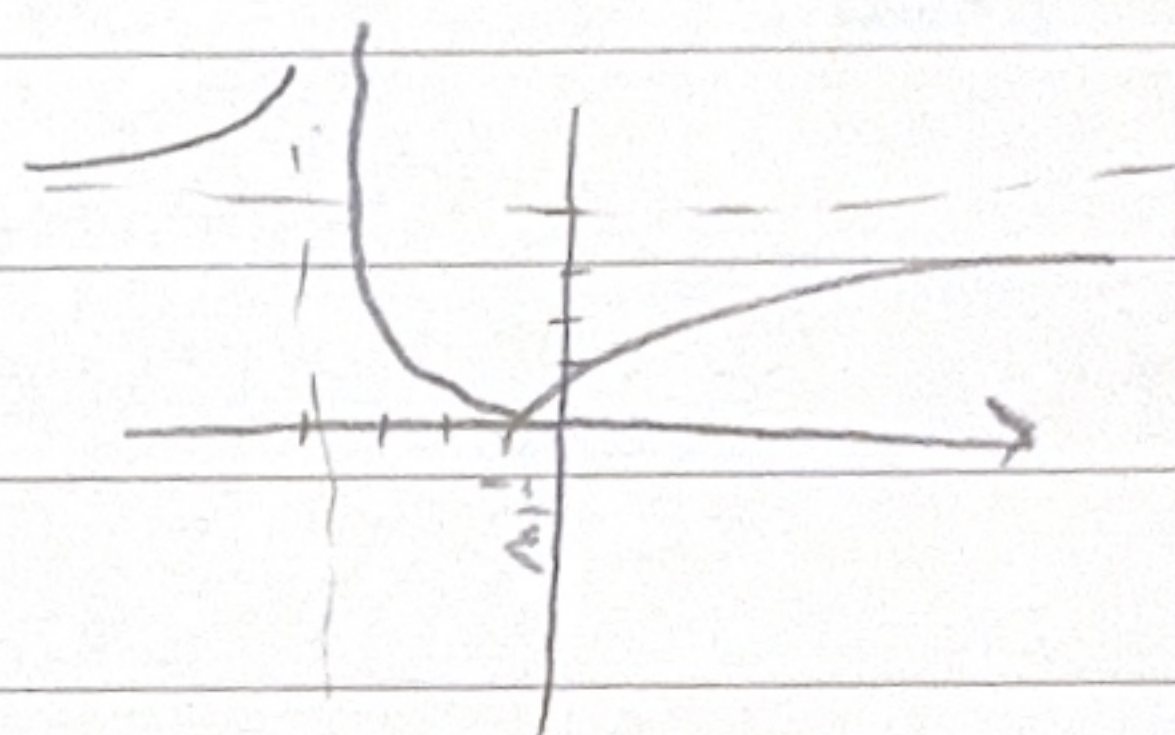


$$y = \left| \frac{r n + 1}{n + r} \right|$$

$$y = \frac{r n + 1}{n + r}$$

$$\frac{r n + 1}{n + r} = 0 \rightarrow n = -\frac{1}{r}$$

در نقطه
مماس



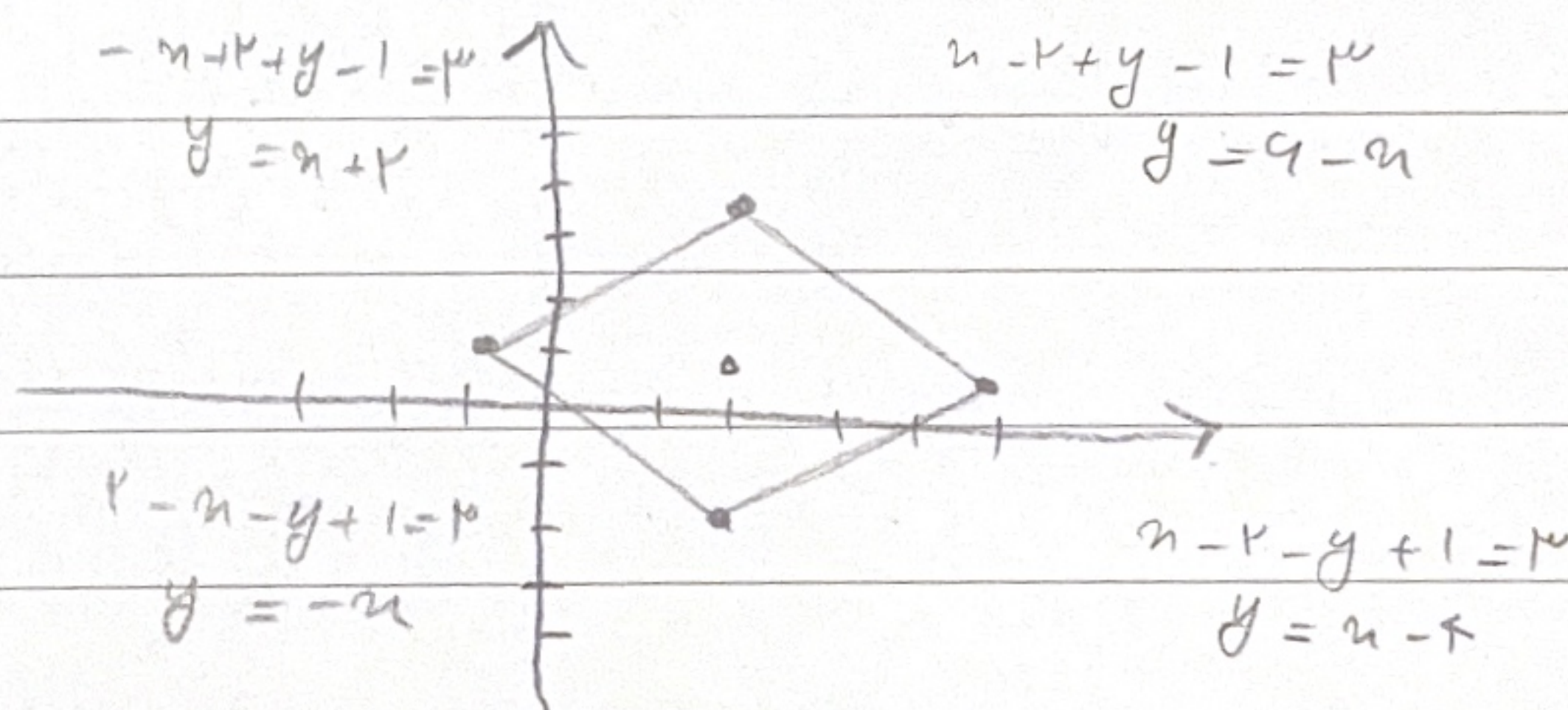
$$|n - r| + |y - 1| = r$$

$$-n - r + y - 1 = r \quad y = n + r$$

$$n - r + y - 1 = r \quad y = r - n$$

$$r - n - y + 1 = r \quad y = -n$$

$$n - r - y + 1 = r \quad y = n - r$$



$$\text{ب) } f(n) = n + \frac{1}{n} + r \sqrt{n + \frac{1}{n}} \quad n > 0 \quad n + \frac{1}{n} > r$$

$$f(n) = \left(\sqrt{n + \frac{1}{n}} \right)^2 + r \sqrt{n + \frac{1}{n}} \Rightarrow \sqrt{n + \frac{1}{n}} \quad \frac{b}{a} = \frac{r}{1} = r$$

$$R_f = [r + r\sqrt{r}, +\infty)$$

$$\rightarrow y = (r - \sin n)(1 + \sin n) \quad y = -\sin^2 n + r \sin n + r \Rightarrow \sin n \quad \frac{b}{a} = \frac{r}{-1} = -r$$

$$R_g = [0, r]$$

$$f(n) = -\frac{1}{p}n + 10 \quad D_f = \{1, \dots, 9\}$$

$$= -\frac{1}{p} \times 1 + 10 + -\frac{1}{p} \times 2 + 10 + \dots + -\frac{1}{p} \times 9 + 10 = -\frac{1}{p} (1+2+\dots+9) + 9 \times 10 = -\frac{1}{p} \times 45 + 90$$

$$= \frac{9 \times 10}{p} = 90$$

$$-10 + 10 = \boxed{0}$$

$$f(n) = \sqrt{an^2 + bn + c} \rightarrow D_f = [K, +\infty) \quad a=0 \rightarrow f(n) = \sqrt{bn+c}$$

$$x-1 \mid \begin{cases} rb+c=0 \\ rb+c=1 \end{cases}$$

$$a=0 \quad b=-1 \quad c=-r$$

$$\begin{cases} rb+c=0 \\ rb+c=1 \end{cases} \rightarrow \begin{cases} b=-1 \\ c=-r \end{cases}$$

$$f(n) = \sqrt{bn^2 + an + c} \rightarrow g(n) = \sqrt{n^2 + r} \rightarrow y' = n^2 + r \quad \text{ext} \mid \begin{cases} \frac{-b}{2a} = 0 \\ y = r \end{cases}$$

$$Ry' = [K, +\infty) \rightarrow Rg = \sqrt{[K, +\infty)} \rightarrow [K, +\infty)$$

$$f(n) = \frac{n^p - rn}{n+r} \Rightarrow \frac{n(n^p - r)}{n+r} \rightarrow \frac{n(n-r)(n+r)}{n+r} \rightarrow f(n) = n^p - rn$$

$$f(n) = -r \rightarrow f(-r) = (-r)^p - r \times -r = 1 \quad n^p - rn = 1 \quad (n+r)(n-r) = 0$$

$$Rf(n) \Rightarrow Rf(n) = n^p - rn \rightarrow \text{ext} \mid \begin{cases} n = -\frac{b}{2a} = 1 \\ y = 1 - r \times 1 = -1 \end{cases} \rightarrow D_f = [R - \sum -r, K] \quad \begin{cases} a = -r \\ b = r \end{cases}$$

$$y = \sqrt[p]{n^p + an^p + bn - 1} \xrightarrow{\text{Gib} \cdot r} y = \sqrt[p]{(n-r)^p} \rightarrow y = \sqrt[p]{n^p - 2rn^p + 14n - 1} \quad \begin{matrix} a = -r \\ b = r \end{matrix}$$

$$Dy = [a, b] \rightarrow Dy = [-4, 14] \rightarrow -4 \leq n \leq 14 \rightarrow -1 \leq n-r \leq 10$$

$$Rf = [-1, 10] \quad y = \sqrt[p]{(n-r)^p} \rightarrow y = n-r \leftarrow$$

$$y = [n] + [n + \frac{1}{p}]$$

$$\frac{1}{p} \leq n < 1 \rightarrow 1 \leq n + \frac{1}{p} < \frac{p}{p} \Rightarrow [n] = 0 \quad [n + \frac{1}{p}] = 1 \rightarrow 0 + 1 = 1$$

$$1 \leq n < \frac{p}{p} \rightarrow \frac{p}{p} \leq n + \frac{1}{p} < 2 \Rightarrow [n] = 1 \quad [n + \frac{1}{p}] = 1 \rightarrow 1 + 1 = 2$$

$$\frac{p}{p} \leq n < 2 \rightarrow 2 \leq n + \frac{1}{p} < 2 \Rightarrow [n] = 1 \quad [n + \frac{1}{p}] = 2 \rightarrow 1 + 2 = 3$$

$$1 \leq n < 2$$

