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(الف) $f(n) = 2 \sin^2(n + \frac{\pi}{4}) - 1 \rightarrow -1 \leq \sin n \leq 1 \rightarrow 0 \leq \sin^2 n \leq 1 \rightarrow$

$0 \leq 2 \sin^2 n \leq 2 \rightarrow -1 \leq 2 \sin^2 n - 1 \leq 1 \Rightarrow R_f = [-1, 1]$

(ب) $g(n) = 3 \cos^3 n - 4 \rightarrow -1 \leq \cos n \leq 1 \rightarrow -1 \leq \cos^3 n \leq 1 \rightarrow -3 \leq 3 \cos^3 n \leq 3$

$\rightarrow -7 \leq 3 \cos^3 n - 4 \leq -1 \Rightarrow R_g = [-7, -1]$

$y = \frac{n}{|n|} + [n]$
در این تابع نسبت

$(0, 1) \rightarrow [n] = 0 \rightarrow y = \frac{n}{|n|} \xrightarrow{n > 0} y = \frac{n}{n} = 1$

$[1, 2) \rightarrow [n] = 1 \Rightarrow y = \frac{n}{|n|} + 1 \xrightarrow{n > 0} y = 1 + 1 = 2$

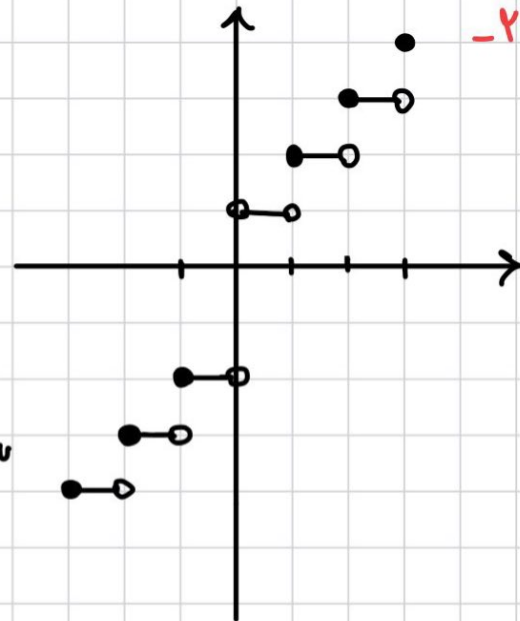
$[2, 3) \rightarrow [n] = 2 \Rightarrow y = \frac{n}{|n|} + 2 \xrightarrow{n > 0} y = 1 + 2 = 3$

$3 \rightarrow [n] = 3 \Rightarrow y = \frac{n}{|n|} + 3 \xrightarrow{n > 0} y = 1 + 3 = 4$

$[-1, 0) \rightarrow [n] = -1 \rightarrow y = \frac{n}{|n|} - 1 \xrightarrow{n < 0} -1 - 1 = -2$

$[-2, -1) \rightarrow [n] = -2 \rightarrow y = \frac{n}{|n|} - 2 \xrightarrow{n < 0} -1 - 2 = -3$

$[-3, -2) \rightarrow [n] = -3 \rightarrow y = \frac{n}{|n|} - 3 \xrightarrow{n < 0} -1 - 3 = -4$

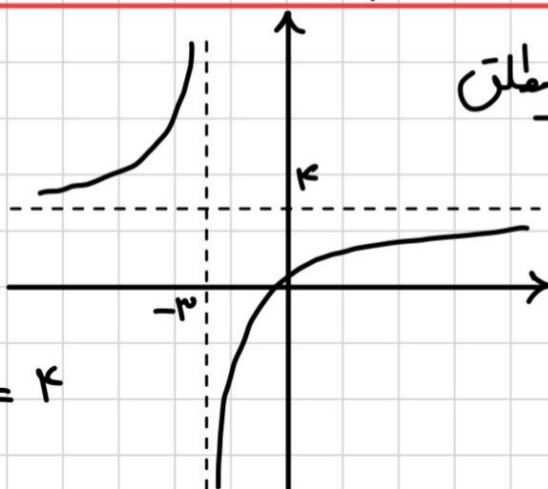


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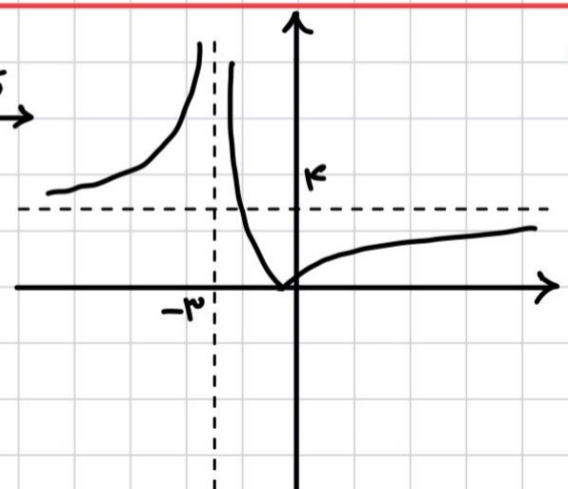
$y = \left| \frac{kx+1}{x+k} \right|$

مجاانب قائم = $-k$

مجاانب افقی = $\frac{a}{c} = k$



قدر مطلق



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$$y = |x-2| + |y-1| = 3$$

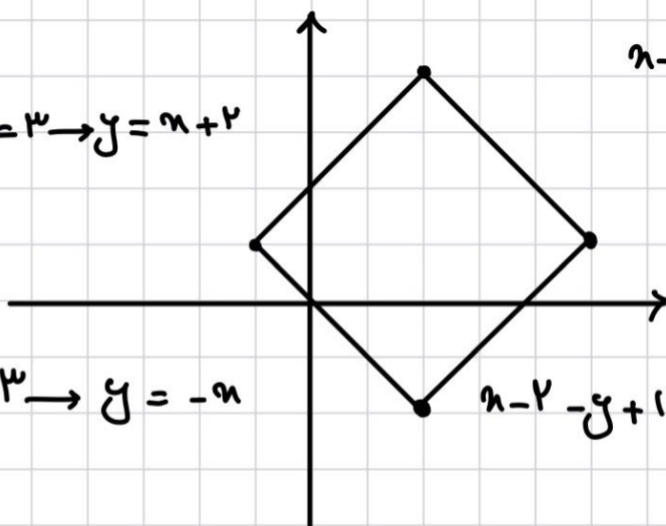
$$-x+2+y-1=3 \rightarrow y=x+2$$

$$x-2+y-1=3 \Rightarrow$$

$$y=-x+6$$

$$-x+2-y+1=3 \rightarrow y=-x$$

$$x-2-y+1=3 \rightarrow y=x-4$$



الف) $f(x) = x + \frac{1}{x} + 2\sqrt{x + \frac{1}{x}} \rightarrow \left(\sqrt{x + \frac{1}{x}}\right)^2 + 2\sqrt{x + \frac{1}{x}} \xrightarrow{\max} +\infty$

$$R_f = [2 + 2\sqrt{2}, +\infty)$$

$$\begin{aligned} \min & \quad x + \frac{1}{x} = 2 \rightarrow 2 + 2\sqrt{2} \\ \frac{-b}{2a} & = \frac{-1}{2} = -\frac{1}{2} \Rightarrow \end{aligned}$$

ب) $y = (3 - \sin x)(1 + \sin x) = -\sin^2 x + 2\sin x + 3 \xrightarrow{\max} \sin x = 1 \rightarrow y = 4$

$$R_f = [0, 4]$$

$$\begin{aligned} \min & \quad \sin x = -1 \rightarrow y = 0 \\ \frac{-b}{2a} & = 1 \end{aligned}$$

$$f(x) = -\frac{1}{3}x + 10 \rightarrow D_f = \{1, 2, 3, \dots, 9\}$$

$$f(1) = -\frac{1}{3}(1) + 10, f(2) = -\frac{1}{3}(2) + 10, \dots, -\frac{1}{3}(9) + 10 \Rightarrow f(1) + f(2) + \dots + f(9) =$$

$$-\frac{1}{3}(1) + 10 + -\frac{1}{3}(2) + 10 + \dots + -\frac{1}{3}(9) + 10 = -\frac{1}{3}(1+2+3+\dots+9) + 9 \times 10 = -\frac{1}{3}(45) + 90 = \frac{82.5}{\sqrt{2}}$$

چون دانه بازه باز و دارای یک عدد $a=0$ است تابع درجه یک است $\Rightarrow$

می دانیم کمترین مقدار را در یکال صفری با $\Rightarrow f(2) = 0 \Rightarrow \sqrt{2b+C} = 0 \Rightarrow 2b+C = 0$

$$f(r) = 1 \Rightarrow \sqrt{rb+C} = 1 \Rightarrow \left. \begin{matrix} rb+C=1 \\ rb+C=0 \end{matrix} \right\} \Rightarrow b=1, C=-r$$

$$g(n) = \sqrt{bn^r + an - rC} \Rightarrow g(n) = \sqrt{n^r + r} \rightarrow \frac{-b}{ra} = 0 \Rightarrow g(0) = r \Rightarrow Rg = [r, +\infty)$$

$$f(n) = \frac{n^r - rn}{n+r} = \frac{n(n^r - r)}{n+r} = \frac{n(n-r)(\cancel{n+r})}{\cancel{n+r}} = n(n-r)$$

$$\hookrightarrow n+r \neq 0 \Rightarrow n \neq -r$$

$$\text{نقطة} \rightarrow n(n-r) \neq 1 \Rightarrow n^r - rn - 1 \neq 0 \Rightarrow n \neq r, -r \text{ II} \Rightarrow D_f = \mathbb{R} - \{-r, r\} \Rightarrow a = -r, b = r$$

$$f(n) = \frac{n(n-r)}{n^r - rn} \Rightarrow \min_{n^r - rn \rightarrow -\frac{b}{ra} = 1} \left| \frac{1}{-1} \right| \Rightarrow R_f = [-1, +\infty) - \{1\} \Rightarrow C = -1$$

$$f\left(\frac{a+b}{rc}\right) = f(-1) = \underline{r}$$

$$y = \sqrt[n]{n^r + an^r + bn - 1} \xrightarrow[\text{خطی است, } b=1]{\text{علاقه}} y = \sqrt[n]{(n-r)^r} \Rightarrow y = \sqrt[n^r - rn^r + rn - 1]$$

$$\Rightarrow a = -r, b = 1r \Rightarrow D_f = [-r, 1r] \rightarrow -r \leq n \leq 1r \rightarrow -1 \leq n-r \leq 1 \Rightarrow$$

$$D_f = [-1, 1]$$

$$y = [n] + \left[n + \frac{1}{r}\right]$$

$$\frac{1}{r} \leq n < 1 \rightarrow [n] = 0 \rightarrow 1 \leq n + \frac{1}{r} < \frac{r}{r} \Rightarrow \left[n + \frac{1}{r}\right] = 1 \rightarrow 0 + 1 = 1$$

$$1 \leq n < \frac{r}{r} \rightarrow [n] = 1 \rightarrow \frac{r}{r} \leq n + \frac{1}{r} < 2 \Rightarrow \left[n + \frac{1}{r}\right] = 1 \rightarrow 1 + 1 = 2$$

$$\frac{r}{r} \leq n < 2 \rightarrow [n] = 1 \rightarrow 2 \leq n + \frac{1}{r} < \frac{2r}{r} \Rightarrow \left[n + \frac{1}{r}\right] = 2 \rightarrow 1 + 2 = 3$$

بجای $\frac{r}{r}$

