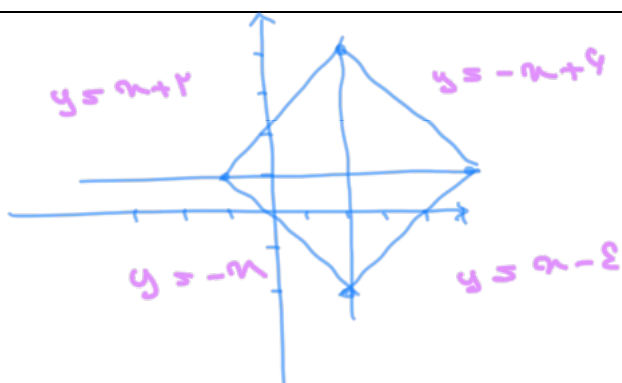
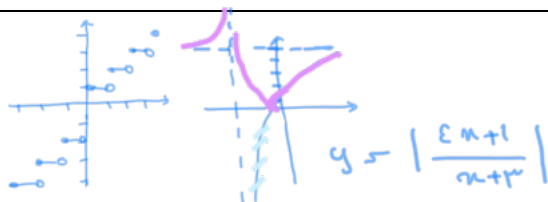


الف) $f(x) = r \sin^2(x + \frac{\pi}{4}) - 1 \rightarrow -1 \leq \sin(x + \frac{\pi}{4}) \leq 1 \Rightarrow 0 \leq \sin^2(x + \frac{\pi}{4}) \leq 1$
 $\Rightarrow 0 \leq r \sin^2(x + \frac{\pi}{4}) \leq 2 \Rightarrow -1 \leq r \sin^2(x + \frac{\pi}{4}) - 1 \leq 1 \Rightarrow R_f = [-1, 1]$

ب) $g(x) = 3 \cos^3 x - 4 \rightarrow -1 \leq \cos x \leq 1 \Rightarrow -1 \leq \cos^3 x \leq 1 \Rightarrow$
 $-3 \leq 3 \cos^3 x \leq 3 \Rightarrow -7 \leq 3 \cos^3 x - 4 \leq -1 \Rightarrow R_g = [-7, -1]$

$-3 \leq x \leq 2 \Rightarrow |x| = -x \Rightarrow \frac{x}{|x|} = -1 \rightarrow y = -x, [x] = -3$
 $-2 \leq x < -1 \Rightarrow \frac{x}{|x|} = 1 \rightarrow y = x, [x] = -2$
 $-1 \leq x < 0 \Rightarrow \frac{x}{|x|} = -1 \rightarrow y = -x, [x] = -1$
 $0 \leq x < 1 \Rightarrow [x] = 0$
 $1 \leq x < 2 \Rightarrow [x] = 1$
 $2 \leq x < 3 \Rightarrow [x] = 2$
 $\rightarrow x = 3 \Rightarrow y = 3$



ج) $f(x) = x + \frac{1}{x} + 2 \sqrt{x + \frac{1}{x}} \rightarrow x + \frac{1}{x} \geq 0 \rightarrow x > 0 \Rightarrow x + \frac{1}{x} \geq 2$
 $\Rightarrow \left(\sqrt{x + \frac{1}{x}} \right)^2 + 2 \sqrt{x + \frac{1}{x}} \rightarrow R_f = [2 + 2\sqrt{2}, +\infty)$

د) $y = (3 - \sin x)(1 + \sin x) = 3 + 3 \sin x - \sin x - \sin^2 x = -\sin^2 x + 2 \sin x + 3$
 $\sin x \rightarrow \begin{matrix} -1 & \rightarrow & -1 - 2 + 3 = 0 \\ 1 & \rightarrow & -1 + 2 + 3 = 4 \\ \frac{1}{2} & \rightarrow & -\frac{1}{4} - 1 + 3 = \frac{5}{4} \end{matrix}$

$$D_f = \{1, r, r^2, \varepsilon, \omega, r^3, r^4, r^5, r^6, r^7, r^8, r^9\} \quad \frac{9 \times 10}{r} = \varepsilon \omega$$

$$-\frac{1}{r} \times 1 + 10 + \dots + \frac{1}{r} \times 9 + 10 = \frac{1}{r} (1 + r + \dots + r^9) + 9 \times 10 = 90 - 10 = \boxed{80}$$

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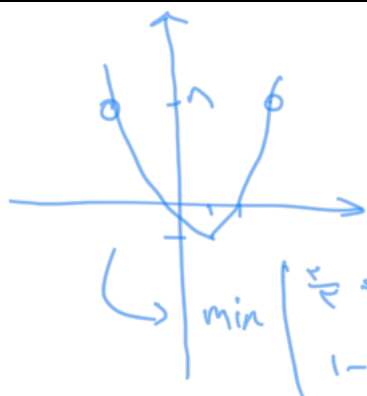
$$f(x) = \sqrt{ax^2 + bx + c} > 0 \rightarrow f(x) = \sqrt{x^2 + 1} \rightarrow x^2 + 1 > 0$$

$$g(x) = \sqrt{x^2 + \varepsilon} \rightarrow \min \left| \varepsilon \right|$$

$$R_g = \sqrt{[\varepsilon, +\infty)} = [r, +\infty)$$

$$\rightarrow \begin{matrix} b=1 \\ c=-r \end{matrix}$$

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$$f(x) = \frac{x^2 - \varepsilon x}{x + r} = \frac{x(x - \varepsilon)(x + r)}{(x + r)} \rightarrow x^2 - \varepsilon x = 1$$

$$\rightarrow x^2 - \varepsilon x - 1 = 0 \rightarrow x = \varepsilon, -1$$

$$\{a, b\} = \{-1, \varepsilon\} \quad f\left(\frac{a+b}{rc}\right) = f(-1) = 1 + r = \mu$$

$$R_f = [-1, +\infty)$$

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$$y = \sqrt[n]{n^2 + an + b} - 1 \xrightarrow{\text{lim}} y = \sqrt[n]{(n-r)^2} = \sqrt[n]{n^2 - 4nr + 4r^2} - 1$$

$$\Rightarrow a = -4, b = 4 \Rightarrow -4 \leq n \leq 14 \Rightarrow -1 \leq n - r \leq 16$$

$$\rightarrow R_f = [1, 1.6]$$

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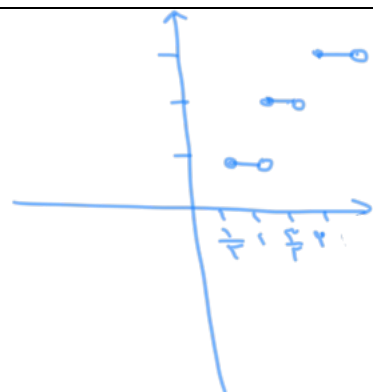
$$y = [x] + [x + \frac{1}{r}]$$

$$\frac{1}{r} \leq x < 1 \rightarrow 1 \leq x + \frac{1}{r} < \frac{r}{r} \rightarrow y = 0 + 1 = 1$$

$$1 \leq x < \frac{r}{r} \rightarrow \frac{r}{r} \leq x + \frac{1}{r} < r \rightarrow y = 1 + 1 = 2$$

$$\frac{r}{r} \leq x < r \rightarrow r \leq x + \frac{1}{r} < \frac{r^2}{r} \rightarrow y = 1 + r = r$$

booby



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