

الف) $2 \sin^2(n + \frac{\pi}{6}) - 1$ ب) $3 \cos^2 n - 2$

$0 \leq \sin^2(n + \frac{\pi}{6}) \leq 1$ $-1 \leq \cos^2 n \leq 1$

$0 \leq 2 \sin^2(n + \frac{\pi}{6}) \leq 2$ $-2 \leq 3 \cos^2 n \leq 3$

$-1 \leq 2 \sin^2(n + \frac{\pi}{6}) - 1 \leq 1$ $-5 \leq 3 \cos^2 n - 2 \leq -1$

$-1 \leq y \leq 1 \Rightarrow R = [-1, 1]$ $-5 \leq y \leq -1$

$R = [-5, -1]$

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$y = \frac{n}{n+1} + [n]$

$[0, 1) \rightarrow 1 + 0 = 1$ $[-1, 0) \rightarrow -1 + (-1) = -2$

$[1, 2) \rightarrow 1 + 1 = 2$ $[-2, -1) \rightarrow -1 + (-2) = -3$

$[2, 3) \rightarrow 1 + 2 = 3$ $[-3, -2) \rightarrow -1 + (-3) = -4$

$\{3\} \rightarrow 1 + 3 = 4$

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$y = \frac{n+1}{n+3}$ $y = \left| \frac{n+1}{n+3} \right|$

محدوده $y = 1$ محدوده $y = -1$

$\frac{n \rightarrow \infty}{y \rightarrow 1} \Rightarrow y = \frac{1}{1} = 1$ $\frac{n \rightarrow -3}{y \rightarrow -\infty}$

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$x - n + y - 1 = 1 \Rightarrow y = 2 - x + 1$

$\Rightarrow y = x + 2$

$x - n + 1 - y = 1 \Rightarrow y = x - n$

$\Rightarrow y = -\infty$

② $(1 - \sin n)(1 + \sin n) =$
 $= (-1)(\sin n - \sin n - 1)$
 $= -\sin^2 n + \sin n + 1$
 $\min \sin n \xrightarrow{1} y = 0$
 $\max \sin n \xrightarrow{1} y = 1$
 $\frac{-b}{2a} = 1 \rightarrow$
 $R = [0, 1]$

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الف) $f(n) = n + \frac{1}{n} + \sqrt{n+1} + 1 - 1 \Rightarrow f(n) = \frac{(\sqrt{n+1} + 1)^2 - 1}{t}$

min: $\min n + \frac{1}{n} \xrightarrow{(2,5)} 2 \Rightarrow \min f = \sqrt{2} + 1$

Max: $\max n + \frac{1}{n} \xrightarrow{(2,5)} +\infty \Rightarrow \max f = +\infty$

$\frac{-b}{2a} : 0 \Rightarrow \sqrt{n+1} + 1 = 0 \Rightarrow \sqrt{n+1} = -1 \Rightarrow$

$\Rightarrow R_f = [\sqrt{2} + 1, +\infty)$

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$$D_f = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

$$n=1 \rightarrow y=1-\frac{1}{1} \quad n=2 \rightarrow y=1-\frac{1}{2} \quad n=3 \rightarrow y=1-\frac{1}{3}$$

$$n=4 \rightarrow y=1-\frac{1}{4} \quad n=5 \rightarrow y=1-\frac{1}{5} \quad n=6 \rightarrow y=1-\frac{1}{6}$$

$$n=7 \rightarrow y=1-\frac{1}{7} \quad n=8 \rightarrow y=1-\frac{1}{8} \quad n=9 \rightarrow y=1-\frac{1}{9}$$

$$1 \cdot \left(\frac{1}{1}\right) + 1 \cdot \left(\frac{1}{2}\right) + 1 \cdot \left(\frac{1}{3}\right) + 1 \cdot \left(\frac{1}{4}\right) + 1 \cdot \left(\frac{1}{5}\right) + 1 \cdot \left(\frac{1}{6}\right) + 1 \cdot \left(\frac{1}{7}\right) + 1 \cdot \left(\frac{1}{8}\right) + 1 \cdot \left(\frac{1}{9}\right) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} = 2.8289682539682539$$

$$A: an^2 + bn + c \quad \frac{n}{A} = \frac{1}{1} + \Rightarrow \frac{a}{b} = 0 \sim bn + c = 0 \quad n = -\frac{c}{b} = 1 \Rightarrow c = -b$$

$$f(n) = 1 \rightarrow \sqrt{bn - 1b} = \sqrt{1b - 1b} = \sqrt{0} = 0 \Rightarrow b = 1, c = -1$$

$$g(n) = \sqrt{2n^2 + 1} \rightarrow 2n^2 + 1 \in [\varepsilon, +\infty) \quad \sqrt{[\varepsilon, +\infty)} = [\sqrt{\varepsilon}, +\infty)$$

$$R(f) = [R - \{ -1 \}]$$

$$f(n) = \frac{n^2 - 1n}{n + 1} = \frac{n(n - 1)}{n + 1} = n(n - 1) = n^2 - n$$

$$\frac{n^2 - 1n}{n + 1} = 1$$

$$n^2 - 1n = 1n + 1$$

$$n^2 - 1n - 1 = 0$$

$$n(n - 1) = 1 \Rightarrow n = 2 \Rightarrow a = -1, b = 1 \Rightarrow a + b = 0$$

$$f\left(\frac{a+b}{c}\right) = f(-1) = (-1)^2 - 1(-1) = 1 + 1 = 2$$

$$\frac{-b}{1a} = \frac{1}{1} = 1$$

$$n = 1 \rightarrow y = -1$$

$$R = [1, +\infty)$$

$$\Rightarrow 1c = -1$$

$$y = \sqrt[n]{A} \xrightarrow{\text{dici}} A = t^n, t = an + b \Rightarrow y = \sqrt[n]{(an + b)^n} = an + b$$

$$n^2 + an^2 + bn - 1 = t^n = (n - 1)^n \Rightarrow n^2 - 2n^2 + 1n - 1 \Rightarrow \begin{cases} a = -1 \\ b = 1 \end{cases}$$

$$-1 \leq n \leq 1 \rightarrow -1 \leq n - 1 \leq 0 \quad (n - 1) \in [-1, 0] \quad R = [-1, 0]$$

$$\left(\frac{1}{1}, 1\right) \rightarrow 0 + 1 = 1$$

$$\left(1, \frac{1}{1}\right) \rightarrow 1 + 1 = 2$$

$$\left(1, \frac{1}{1}\right) \rightarrow 1 + 1 = 2$$

$$\left(\frac{1}{1}, 1\right) \rightarrow 1 + 1 = 2$$

