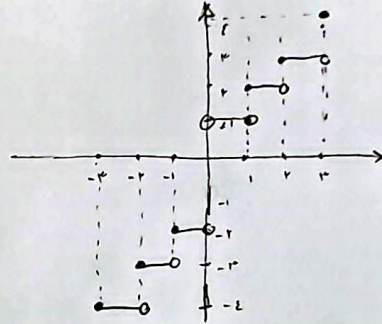


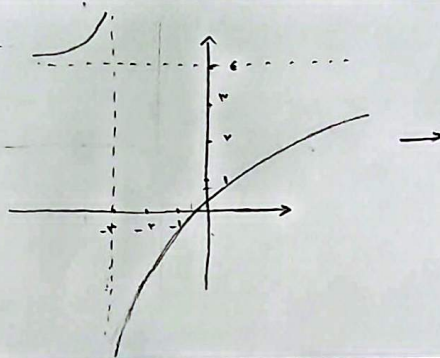
الف) $x + \frac{x}{2} = x \rightarrow 0 \leq \sin^2 x \leq 1 \Rightarrow 0 \leq 2 \sin^2 x \leq 2 \Rightarrow -1 \leq 2 \sin^2 x - 1 \leq 1$
 $\Rightarrow R_f = [-1, 1]$

ب) $-1 \leq \cos x \leq 1 \Rightarrow -1 \leq \cos^2 x \leq 1 \Rightarrow -3 \leq 3 \cos^2 x \leq 3$
 $\rightarrow -7 \leq 3 \cos^2 x - 4 \leq -1 \rightarrow R_g = [-7, -1]$

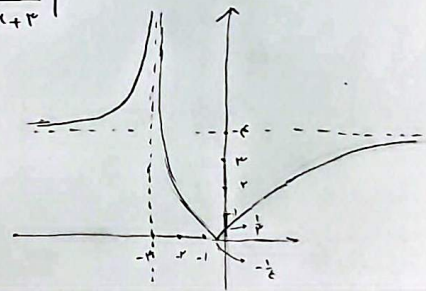
$-3 \leq x < -2 \rightarrow -1 - 3 = -4 = y$
 $-2 \leq x < -1 \rightarrow y = -1 - 2 = -3$
 $-1 \leq x < 0 \rightarrow y = -1 - 1 = -2$
 $0 \leq x < 1 \rightarrow y = 1 + 0 = 1$ (به ازای ۰ تعریف نشده)
 $1 \leq x < 2 \rightarrow y = 1 + 1 = 2$
 $2 \leq x < 3 \rightarrow y = 1 + 2 = 3$
 $x = 3 \rightarrow y = 1 + 3 = 4$



$y = \frac{x+1}{x+3}$
 جنبه قائم = -3
 جنبه افقی = 1
 $x=0 \rightarrow y = \frac{1}{3}$

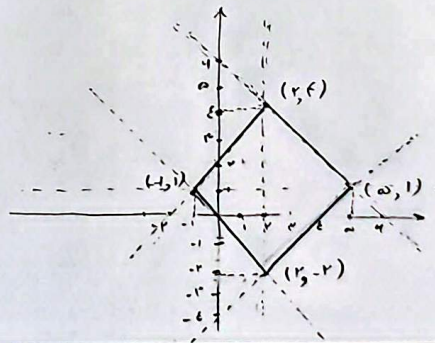


$y = \left| \frac{x+1}{x+3} \right|$



$x \geq 2, y \geq 1 \rightarrow x - 2 + y - 1 = 3 \rightarrow y = 4 - x$
 $x \geq 2, y \leq 1 \rightarrow x - 2 + 1 - y = 3 \rightarrow y = x - 4$
 $x \leq 2, y \geq 1 \rightarrow 2 - x + y - 1 = 3 \rightarrow y = x + 2$
 $x \leq 2, y \leq 1 \rightarrow 2 - x + 1 - y = 3 \rightarrow y = -x$

(خطوط متقاطع از خطوط مستقیم و منحنی)



الف) $\sqrt{x + \frac{1}{x}} = c \rightarrow c^2 + \frac{1}{c} \leq c \rightarrow x + \frac{1}{x} \geq c^2 \rightarrow \sqrt{c^2} \rightarrow c + \frac{1}{c}$
 $\frac{b}{a} \rightarrow \frac{-1}{1} = -1 \rightarrow c \rightarrow 1$

$\rightarrow R_f = [1 + \sqrt{2}, +\infty)$

ب) $y = -\sin^2 x + 2 \sin x + 3 \rightarrow -1 \leq \sin x \leq 1 \rightarrow \sin x = -1 \rightarrow y = 0$
 $\sin x = 1 \rightarrow y = 4 \rightarrow R = [0, 4]$
 $\sin x = \frac{b}{a} = \frac{2}{1} = 2$

$$D_f = \{1, 2, \dots, 9\}$$

$$\rightarrow R_f = \left\{10 - \frac{1}{1}, 10 - \frac{2}{1}, \dots, 10 - \frac{9}{1}\right\}$$

$$\rightarrow \text{مجموع اعداد} : 10 \times 9 - \left(\frac{1+2+\dots+9}{1}\right) = 90 - \frac{(10)(9)}{2} = 90 - 45 = 45$$

6

با توجه به نامندی داده شده تابع صحیحاً تعریف شده است زیرا در هر این صورت دامنه به صورت $[a, b]$ و $\mathbb{R} - (a, b)$ می باشد.

$$\Rightarrow a=0 \rightarrow \begin{cases} bx+c \geq 0 \rightarrow x \geq -\frac{c}{b} \rightarrow -\frac{c}{b}=2 \rightarrow 2b+c=0 \\ f(1)=1 = \sqrt{4b+c} \rightarrow 1=2b+c \rightarrow 4b+c=1 \end{cases} \begin{cases} b=1 \\ c=-2 \end{cases}$$

7

$$g(x) = \sqrt{x^2+4} \rightarrow x^2+4 \rightarrow \text{بردار} : [2, +\infty) \rightarrow R_g = \sqrt{[4, +\infty)} = [2, +\infty)$$

$$D_f = \mathbb{R} - \{-2\} \rightarrow \text{صورت} : x+2 \neq 0 \rightarrow x \neq -2$$

$$\Rightarrow a=b=-2 \rightarrow f(x) = \frac{x(x-2)(x+2)}{x+2} = x(x-2) = x^2-2x \rightarrow f(-2) = 8$$

$$\rightarrow \text{ریشه ها} : \frac{2}{x} = 1 \rightarrow f(1) = -1 \Rightarrow R_f = [-1, +\infty) - \{1\} \rightarrow c = -1$$

$$f\left(\frac{a+b}{2}\right) = f\left(\frac{-2}{2}\right) = f(-1) = 1(-1-1) = -2$$

8

$$y = \sqrt[3]{(x-2)^3} = x-2 \rightarrow \text{تابع خطی}$$

$$\Rightarrow (x-2)^3 = x^3 + 3(-2)(x)(x-2) - 8 = x^3 - 6x^2 + 12x - 8$$

$$\Rightarrow a = -4, b = 12 \rightarrow D = [-4, 12]$$

$$x = -4 \rightarrow y = -4 - 2 = -6$$

$$x = 12 \rightarrow y = 12 - 2 = 10 \Rightarrow R = [-6, 10]$$

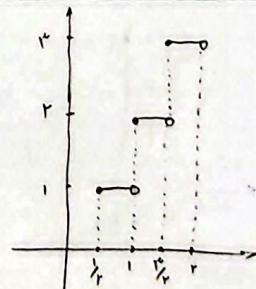
چون تابع ابتدا صعودی است و سپس نزولی دامنه را برای پیدا کردن بردار می دهیم:

$$\frac{1}{2} \leq x < 1 \rightarrow 1 \leq x + \frac{1}{2} < \frac{3}{2} \rightarrow y = 0 + 1 = 1$$

$$1 \leq x < \frac{3}{2} \rightarrow \frac{3}{2} \leq x + \frac{1}{2} < 2 \rightarrow y = 1 + 1 = 2$$

$$\frac{3}{2} \leq x < 2 \rightarrow 2 \leq x + \frac{1}{2} < \frac{5}{2} \rightarrow y = 1 + 2 = 3$$

سپرده خطی



10