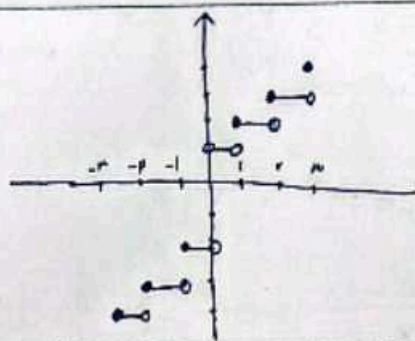


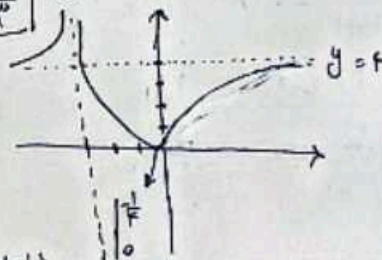
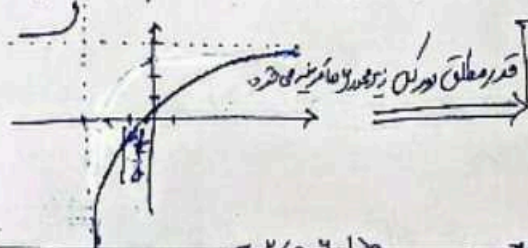
الف) $f(x) = 2\sin(x + \frac{\pi}{4}) - 1$ $-1 \leq \sin x \leq 1 \rightarrow 0 \leq \sin^2(x + \frac{\pi}{4}) \leq 1 \rightarrow 0 \leq 2\sin^2(x + \frac{\pi}{4}) \leq 2$
 $\rightarrow -1 \leq \sin^2(x + \frac{\pi}{4}) \leq 1$ $R_f = [-1, 1]$

ب) $g(x) = 3\cos^3 x - 4$ $-1 \leq \cos x \leq 1 \rightarrow -1 \leq \cos^3 x \leq 1 \rightarrow -3 \leq 3\cos^3 x \leq 3 \rightarrow$
 $-7 \leq 3\cos^3 x - 4 \leq -1$ $R_g = [-7, -1]$

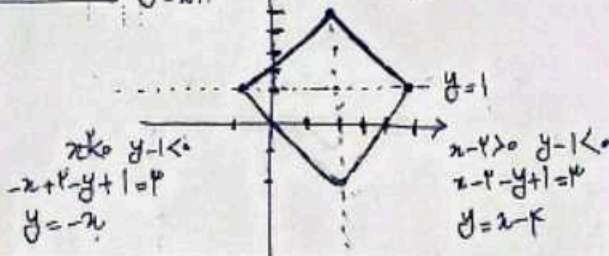
$y = \frac{x}{|x|} + [x]$ $-3 \leq x < -2 \rightarrow y = -1 + (-3) = -4$
 $-2 \leq x < -1 \rightarrow y = -1 + (-2) = -3$
 $-1 \leq x < 0 \rightarrow y = -1 + (-1) = -2$
 $0 \leq x < 1 \rightarrow y = 1 + 0 = 1$
 $1 \leq x < 2 \rightarrow y = 1 + 1 = 2$
 $2 \leq x < 3 \rightarrow y = 1 + 2 = 3$
 $x = 3 \rightarrow y = 1 + 3 = 4$



$y = \left| \frac{x+1}{x+3} \right|$ $\vec{y} = \frac{x+1}{x+3}$ $\vec{y} = \frac{x+1}{x+3}$ $\vec{y} = \frac{x+1}{x+3}$



$|x-2| + |y-1| = 3$ $x-2 < 0, y-1 < 0 \rightarrow -x+2+y-1=3 \rightarrow y=-x+2$
 $x-2 < 0, y-1 > 0 \rightarrow -x+2+y-1=3 \rightarrow y=-x+2$
 $x-2 > 0, y-1 < 0 \rightarrow x-2+y-1=3 \rightarrow y=-x+6$
 $x-2 > 0, y-1 > 0 \rightarrow x-2+y-1=3 \rightarrow y=-x+6$



الف) $f(x) = x + \frac{1}{x} + 2\sqrt{x + \frac{1}{x} + 1} - 1 = \left(x + \frac{1}{x} + 1 \right)^2 - 1 \rightarrow R_f = [0, +\infty)$

ب) $y = (4 - \sin x)(1 + \sin x) = -\sin^2 x + 3\sin x + 4$ $\begin{cases} \rightarrow \sin x = -1 \rightarrow y = -1 - 2 + 4 = 1 \\ \rightarrow \sin x = 1 \rightarrow y = -1 + 2 + 4 = 5 \end{cases}$
 $R_y = [1, 5]$

$$f(x) = \frac{1}{x} x + 0 \quad f(1) = \frac{1}{1} + 0 = \frac{19}{1} \quad f(r) = \frac{19}{r} \quad \dots \quad f(9) = \frac{1}{9} + \frac{19}{9} = \frac{20}{9}$$

$$S = \frac{n}{p} (a+L) = \frac{9}{1} \left(\frac{19}{1} + \frac{20}{9} \right) = \frac{9}{1} \times \frac{199}{9} = \boxed{199}$$

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$$f(x) = \sqrt{ax^2 + bx + c} \quad ax^2 + bx + c \geq 0 \quad \frac{-b}{2a} + \dots \Rightarrow a=0, b>0$$

$$f(x) = \sqrt{bx + c} = 0 \rightarrow bx + c = 0 \rightarrow b = -1, c = 1$$

$$f(x) = \sqrt{bx + c} = 1 \rightarrow bx + c = 1 \rightarrow b = 1, c = 0$$

$$g(x) = \sqrt{x^2 + 0 + 1} = \sqrt{x^2 + 1}$$

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$$R_g = \sqrt{[x_0 + \infty)} = [x_0 + \infty)$$

$$f(x) = \frac{x^2 - 1}{x + 1} \quad \text{find } R_f = \{a, b\} \quad a = -1$$

$$b = ? \quad R_f \text{ exists} \Rightarrow f(b) = 1$$

$$f(b) = \frac{b(b^2 - 1)}{b + 1} = 1 \quad b(b^2 - 1) = b + 1 \quad b = 1$$

$$b = -1$$

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$$f(x) = \frac{x(x-1)(x+1)}{x+1} = x^2 - 1 \quad R_f = [-1, +\infty) \rightarrow C = -1$$

$$\frac{a+b}{pc} = \frac{1}{-1} = -1$$

$$f(-1) = \frac{(-1)^3 - 1(-1)}{-1+1} = \frac{-1+1}{0} = \boxed{0}$$

$$y = \sqrt[3]{x^3 + ax^2 + bx - 1} \quad \frac{1}{x^3 + ax^2 + bx - 1} = (x-1)^3 = x^3 - 3x^2 + 3x - 1$$

$$a = -3, b = 3 \quad x \in [-1, 1]$$

$$y = \sqrt[3]{(x-1)^3} = x-1 \quad -1 \leq x \leq 1 \rightarrow -2 \leq x-1 \leq 0$$

$$R_y = [-2, 0]$$

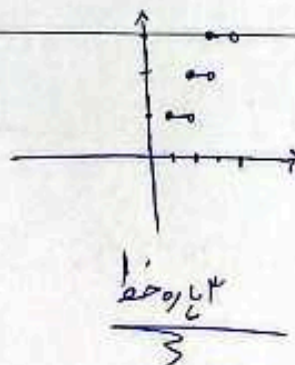
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$$y = [x] + [x + \frac{1}{p}]$$

$$1 \leq x < 2 \rightarrow y = 1 + 1 = 2$$

$$\frac{1}{p} \leq x < 1 \rightarrow y = 0 + 1 = 1$$

$$0 \leq x < \frac{1}{p} \rightarrow y = 0 + 0 = 0$$



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