

دو از دست دوم و دست سوم

مسئله ها

الف) $f(x) = 2 \sin^2(x + \frac{\pi}{4}) - 1$

مسئله

$$0 \leq \sin^2(x + \frac{\pi}{4}) \leq 1 \rightarrow 0 \leq 2 \sin^2(x + \frac{\pi}{4}) \leq 2$$

$$\rightarrow -1 \leq 2 \sin^2(x + \frac{\pi}{4}) - 1 \leq 1$$

$$\rightarrow Rf = [-1, 1]$$

ب) $f(x) = 2 \cos^3 - 1 \rightarrow -1 \leq 2 \cos^3 \leq 1$

$$\rightarrow -\frac{1}{2} \leq \cos^3 \leq \frac{1}{2} \rightarrow -\sqrt[3]{\frac{1}{2}} \leq \cos \leq \sqrt[3]{\frac{1}{2}}$$

$$\rightarrow [-\sqrt[3]{\frac{1}{2}}, \sqrt[3]{\frac{1}{2}}]$$

$y = \frac{x}{|x|} + [x] \quad [-3, 3]$

مسئله

$[-3, -2] \xrightarrow{[x] = -3} -1 - 3 = -4$

$[-2, -1) \xrightarrow{[x] = -2} -1 - 2 = -3$

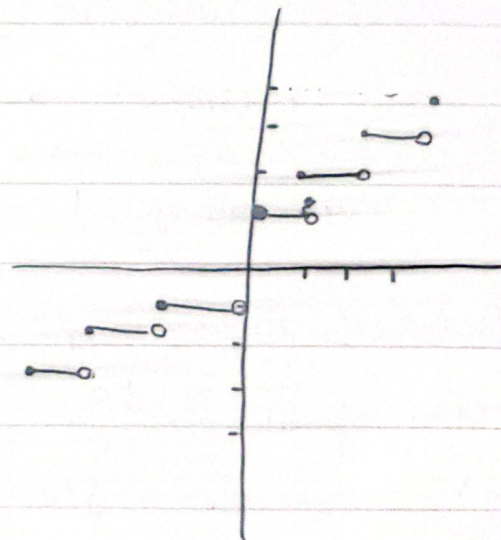
$(-1, 0) \xrightarrow{[x] = -1} -1 - 1 = -2$

$[0, 1) \xrightarrow{[x] = 0} 0 - 1 = -1$

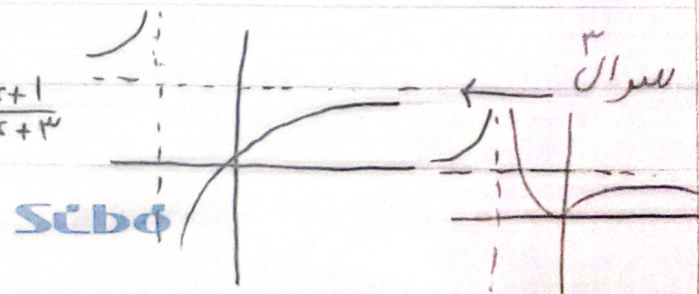
$[1, 2) \xrightarrow{[x] = 1} 1 - 1 = 0$

$[2, 3) \xrightarrow{[x] = 2} 2 - 1 = 1$

$[3, 3] \xrightarrow{[x] = 3} 3 - 1 = 2$

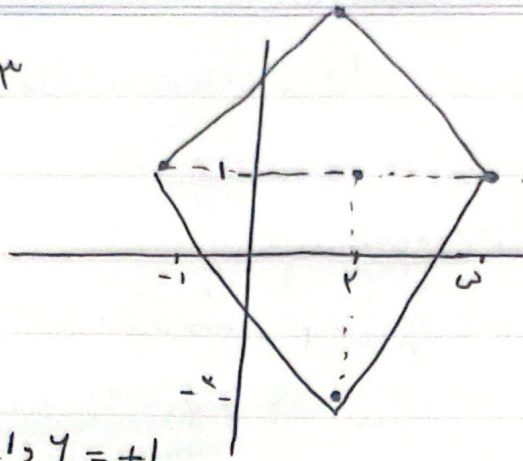


$y = \left| \frac{x+1}{x+3} \right| \rightarrow \frac{x+1}{x+3}$



$$|x-2| + |y-1| = 3$$

سوال ←



$$\textcircled{1} \quad x+y = -x+y \\ \rightarrow x=2, y=1$$

$$\textcircled{2} \quad x+y = -x \rightarrow x=-1, y=+1$$

$$\textcircled{3} \quad -x = x-1 \rightarrow x=2, y=-2$$

$$\textcircled{4} \quad x-1 = -x+1 \rightarrow x=2 \rightarrow x=5, y=1$$

$$\text{الف) } f(x) = x + \frac{1}{x} + x\sqrt{x + \frac{1}{x}}$$

سوال ←

$$x + \frac{1}{x} = a \rightarrow a + 2\sqrt{a} + 1 - 1 \rightarrow (\sqrt{a} + 1)^2 - 1$$

$$\rightarrow (\sqrt{a})(\sqrt{a} + 2) \xrightarrow{\text{کدام مقدار}} a = x + \frac{1}{x} \xrightarrow{x=1} 2$$

$$\rightarrow (\sqrt{2})(\sqrt{2} + 2) \rightarrow Rf = [2 + 2\sqrt{2}, +\infty)$$

$$\text{ب) } y = (3 - \sin x)(1 + \sin x) \rightarrow -\sin^2 x + 2\sin x + 3$$

$$\begin{cases} \sin x = 1 \rightarrow 2 \\ \sin x = -1 \rightarrow 0 \\ \sin x = \frac{1}{2} \rightarrow 1 \end{cases}$$

$$Rf = [0, 3]$$

$$f(x) = -\frac{1}{x} + 1$$

$$D = \{1, 2, 3, \dots, 9\}$$

سوال ←

$$1 \rightarrow -\frac{1}{1} + 1 \rightarrow \frac{0}{1}$$

$$2 \rightarrow -\frac{1}{2} + 1 \rightarrow \frac{1}{2}$$

$$9 \rightarrow -\frac{1}{9} + 1 = \frac{8}{9}$$

$$R = \left\{ \frac{0}{1}, \frac{1}{2}, \dots, \frac{8}{9} \right\}$$

$$\rightarrow \frac{0+1+\dots+8}{9} = \frac{36}{9} = 4$$

$$\rightarrow \boxed{4}$$

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$$f(x) = \sqrt{ax^r + bx + c} \quad D = (r, +\infty) \quad \leftarrow \text{سوال}$$

$$\hookrightarrow a=0 \rightsquigarrow bx + c = 0 \xrightarrow{x=r} rb + c = 0 \rightarrow c = -rb$$

$$\hookrightarrow \sqrt{bx - rb} \quad g(x) = \sqrt{x^r + c}$$

$$\begin{matrix} rb + c = 0 \\ rb + c = 1 \end{matrix} \rightarrow c = -r, b = 1$$

$$\frac{-b}{ra} = -\frac{1}{r} = -\frac{1}{r} \quad \sqrt{(x, +\infty)} = (x, +\infty)$$

$$f(x) = \frac{x^r - [x]}{x+r} = \frac{x(x^r - [x])}{x+r} = \frac{x(x - \frac{1}{r})(x - \frac{1}{r})}{x+r} \quad \leftarrow \text{سوال}$$

$$\rightarrow x^r - \frac{1}{r}x$$

$$D = R - \{a, b\} \quad R = [c, +\infty) - \{a\}$$

$$\rightarrow x^r - \frac{1}{r}x = 1 \rightarrow x^r - \frac{1}{r}x - 1 = (x - \frac{1}{r})(x + \frac{1}{r})$$

$$\rightarrow a = \frac{1}{r}, b = -\frac{1}{r}$$

$$[c, +\infty) \rightarrow x^r - \frac{1}{r}x \rightarrow \frac{-b}{ra} = 1 \rightarrow (1)^r - \frac{1}{r} = -1$$

$$c = -1 \rightarrow f\left(\frac{1-x}{-r}\right) = f(-1) = (-1)^r - \frac{1}{r}(-1) = \frac{1}{r}$$

$$y = \sqrt{x^r + ax^r + bx - 1} \rightarrow a = -r, b = 1 \rightarrow \sqrt{(x - \frac{1}{r})^r} \quad \leftarrow \text{سوال}$$

$$D = \left[\frac{1}{r}, 1\right]$$

$$\rightarrow R_f = [-1, 1]$$

$$y = [x] + \left[x + \frac{1}{r}\right] \quad \left[\frac{1}{r}, 1\right]$$

$$\left[\frac{1}{r}, 1\right] \quad [x] = 0 \quad \left[x + \frac{1}{r}\right] = 1 \quad 0 + 1 = 1$$

$$\left[1, \frac{r}{r}\right] \quad [x] = 1 \quad \left[x + \frac{1}{r}\right] = 1 \quad 1 + 1 = 2$$

$$\left[\frac{r}{r}, \frac{r}{r}\right] \quad [x] = 1 \quad \left[x + \frac{1}{r}\right] = r \quad r + 1 = r$$

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