

الف) $f(u) = r \sin^2(u + \frac{\pi}{2}) - 1$

$0 \leq \sin^2(u + \frac{\pi}{2}) \leq 1 \Rightarrow 0 \leq r \sin^2(u + \frac{\pi}{2}) \leq r$

$\Rightarrow -1 \leq r \sin^2(u + \frac{\pi}{2}) - 1 \leq 1 \quad \{R_f = [-1, 1]\}$

ب) $g(u) = r \cos^2 u - r$

$-1 \leq \cos^2 u \leq 1 \Rightarrow -r \leq r \cos^2 u \leq r \Rightarrow -r \leq r \cos^2 u - r \leq -1$

$\{R_g = [-r, -1]\}$

$y = \frac{x}{|x|} + [x]$

$[-r, r]$

$[-r, -r) \xrightarrow{[x]=r} -1-r = -r$

$[-r, -1) \xrightarrow{[x]=r} -1-r = -r$

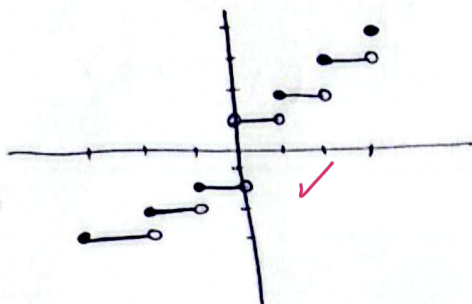
$[-1, 0) \xrightarrow{[x]=r} -1-r = -r$

$[0, 1) \xrightarrow{[x]=r} 1-r = 1$

$[1, r) \xrightarrow{[x]=1} 1+1 = r$

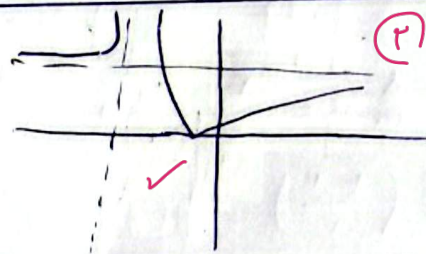
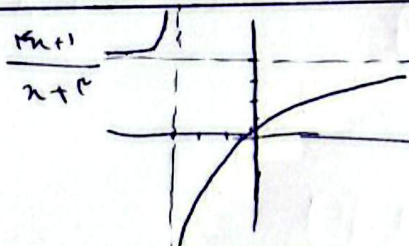
$[r, r) \xrightarrow{[x]=r} 1+r = r$

$\{r\} \xrightarrow{[x]=r} 1+r = r$



نقطة (0, 1) هي نقطة تقاطع

$y = \left| \frac{x+1}{x+r} \right|$



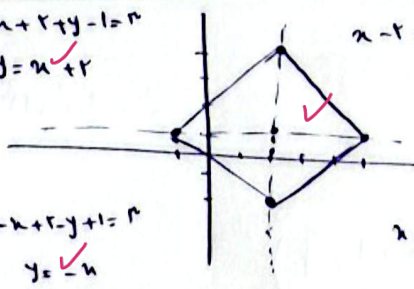
$|x-r| + |y-1| = r$

$-x+r+y-1=r \Rightarrow y=x+r$

$-x+r-y+1=r \Rightarrow y=-x$

$x-r+y-1=r \Rightarrow y=-x+y$

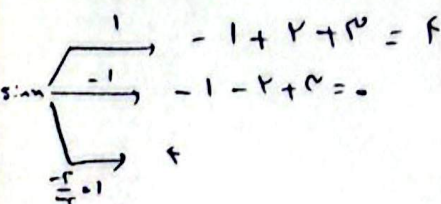
$x-r-y+1=r \Rightarrow y=x-r$



- ① $x+r = -x+y \Rightarrow x \leq r \Rightarrow x \leq r, y \leq 1$
- ② $x+r = -x+y \Rightarrow x \geq r \Rightarrow x \geq r, y \leq 1$
- ③ $-x+r-y+1=r \Rightarrow x \leq r, y \geq 1$
- ④ $x-r-y+1=r \Rightarrow x \geq r, y \geq 1$

و) $f(u) = x + \frac{1}{x} + r \sqrt{x + \frac{1}{x}} = t^r + t + \left\{ \begin{matrix} \xrightarrow{t \rightarrow +\infty} +\infty \\ \xrightarrow{t \rightarrow 0^+} +\infty \\ \xrightarrow{t \rightarrow 1} r+2\sqrt{r} \end{matrix} \right\} R_f = [r+2\sqrt{r}, +\infty)$

ب) $y = (r - \sin u)(1 + \sin u) = r + r \sin u - \sin u - \sin^2 u = -\sin^2 u + r \sin u + r$



$R_y = [0, r+1]$

$$f(n) = -\frac{1}{n} + 1$$

$$1 \rightarrow -\frac{1}{1} + 1 = \frac{r_1}{1}$$

$$2 \rightarrow -\frac{1}{2} + 1 = \frac{r_2}{2}$$

⋮

$$9 \rightarrow -\frac{1}{9} + 1 = \frac{r_9}{9}$$

$$D = \{1, 2, 3, \dots, 9\}$$

$$R = \left\{ \frac{r_1}{1}, \frac{r_2}{2}, \dots, \frac{r_9}{9} \right\}$$

$$\rightarrow \frac{r_1 + \dots + r_9}{9} = \frac{9 \cdot \frac{(1+9)}{2}}{9} = \frac{r_{10}}{9} = \boxed{\frac{10}{9}}$$

(2) (9)

$$f(n) = \sqrt{an^2 + bn + c}$$

$$D = [r, +\infty) \Rightarrow a = 0 \quad f(n) = \sqrt{bn + c}$$

(2) (10)

$$\begin{cases} rb + c = 0 \\ rb + c = 1 \end{cases}$$

$$\begin{cases} rb + c = 1 \\ rb + c = 1 \end{cases}$$

$$b = 1, c = -1$$

$$g(n) = \sqrt{n^2 + 1}$$

$$-\frac{\Delta}{4a} = -\frac{(0 - 14)}{4} = \frac{7}{2}$$

$$R_g = \sqrt{\left[\frac{7}{2}, +\infty\right)} = \boxed{\left[\frac{7}{2}, +\infty\right)}$$

$$f(n) = \frac{x^r - fn}{n+r} = \frac{x(n^r - f)}{n+r} \xrightarrow{n \neq -r} n(n-r) = n^r - fn$$

$$D = R - \{a, b\} \quad R = [c, +\infty) - \{\lambda\}$$

$$c = -\frac{\Delta}{4a} = -\frac{r}{r} = -1$$

$$x^r - fn \neq \lambda \Rightarrow n^r - fn - \lambda \neq 0$$

$$(n-f)(n+r) \neq 0$$

$$\begin{cases} n = f \\ n = -r \end{cases} \Rightarrow a = f, b = -r$$

$$f\left(\frac{r-r}{-r}\right) = f(-1) = (-1)^r - r(-1) = \boxed{\frac{r}{r}}$$

(2) (11)

$$d = \sqrt{x^2 + ax^r + bx - \lambda} = \sqrt{(x-r)^2}$$

$$D = [-4, 12]$$

$$\begin{cases} -4 \rightarrow -\lambda \\ 12 \rightarrow 10 \end{cases} \rightarrow R_f = \boxed{[-\lambda, 10]}$$

$$= \sqrt{x^2 - 4x^r + 12x - \lambda}$$

$$\begin{cases} a = -4 \\ b = 12 \end{cases}$$

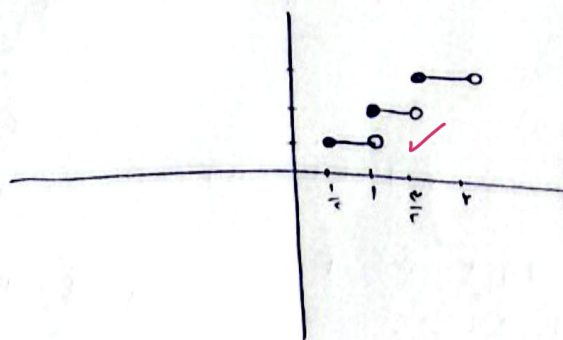
(2) (12)

$$y = [n] + \left[n + \frac{1}{r}\right] \quad \left[\frac{1}{r}, r\right)$$

$$\left[\frac{1}{r}, 1\right) \xrightarrow{[n] \leq 0, [n + \frac{1}{r}] \leq 1} 0 + 1 \leq 1$$

$$\left[1, \frac{r}{r}\right) \xrightarrow{[n] \leq 1, [n + \frac{1}{r}] \leq r} 1 + 1 \leq r$$

$$\left[\frac{r}{r}, r\right) \xrightarrow{[n] \leq r, [n + \frac{1}{r}] \leq r} 1 + r \leq r$$



(2) (13)

✓
b = 1, r