

الف) $f(u) = r \sin^2(u + \frac{\pi}{2}) - 1$

$0 \leq \sin^2(u + \frac{\pi}{2}) \leq 1 \Rightarrow 0 \leq r \sin^2(u + \frac{\pi}{2}) \leq r$

$\Rightarrow -1 \leq r \sin^2(u + \frac{\pi}{2}) - 1 \leq r \quad \{R_f = [-1, r]\}$

ب) $g(u) = r \cos^2 u - r$

$-1 \leq \cos^2 u \leq 1 \Rightarrow -r \leq r \cos^2 u \leq r \Rightarrow -r \leq r \cos^2 u - r \leq -1$

$\{R_g = [-r, -1]\}$

$y = \frac{x}{|x|} + [x]$

$[-r, r]$

$[-r, -r) \xrightarrow{[x]=r} -1-r = -r$

$[-r, -1) \xrightarrow{[x]=r} -1-r = -r$

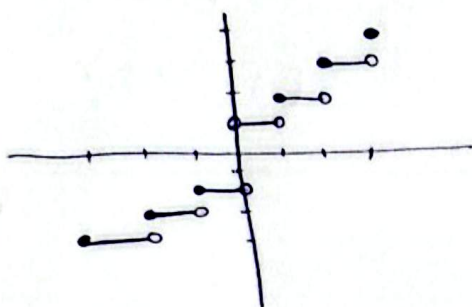
$[-1, 0) \xrightarrow{[x]=r} -1-r = -r$

$[0, 1) \xrightarrow{[x]=r} 1-r = 1$

$[1, r) \xrightarrow{[x]=1} 1+1=r$

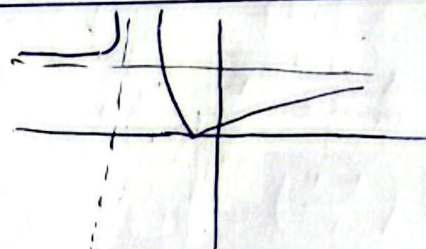
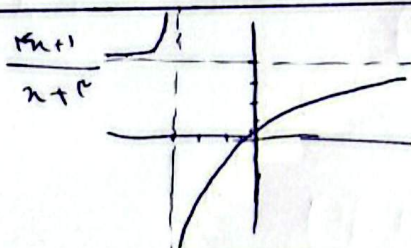
$[r, r) \xrightarrow{[x]=r} 1+r=r$

$\{r\} \xrightarrow{[x]=r} 1+r=r$



نقطة (0,1) هي نقطة انقطاع

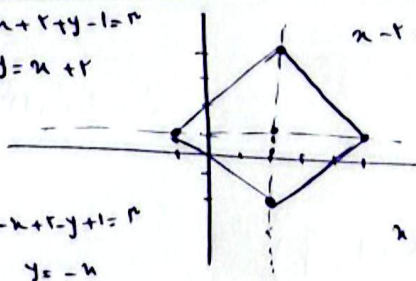
$y = \left| \frac{x+1}{x+r} \right|$



$|x-r| + |y-1| = r$

$-x+r+y-1=r$
 $y=x+r$

$-x+r-y+1=r$
 $y=-x$



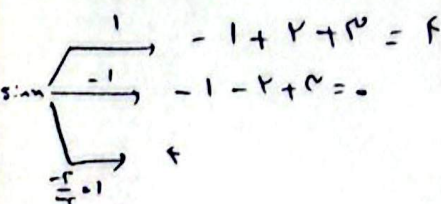
$x-r+y-1=r \Rightarrow y=-x+y$

$x-r-y+1=r$
 $y=x-r$

- ① $x+r = -x+y \rightarrow x \leq r \rightarrow x \leq r, y \leq 1$
- ② $x+r = -x+y \rightarrow x \geq r \rightarrow x \geq r, y \leq 1$
- ③ $-x \leq -x \rightarrow x \leq r$
- ④ $x-r = -x+y \rightarrow x \leq r$
- ⑤ $x-r = -x+y \rightarrow x \geq r$
- ⑥ $x-r = -x+y \rightarrow x \leq r$
- ⑦ $x-r = -x+y \rightarrow x \geq r$
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- ㊿ $x-r = -x+y \rightarrow x \leq r$

و) $f(u) = x + \frac{1}{x} + r \sqrt{x + \frac{1}{x}} = t^r + t + \sqrt{t} \quad R_f = [r + r\sqrt{r}, +\infty)$

ب) $y = (r - \sin u)(1 + \sin u) = r + r \sin u - \sin u - \sin^2 u = -\sin^2 u + r \sin u + r$



$R_y = [0, r]$

$$f(n) = -\frac{1}{n} + 1.$$

$$D = \{1, 2, 3, \dots, 9\}$$

(9)

$$1 \rightarrow -\frac{1}{1} + 1 = \frac{r_1}{1}$$

$$2 \rightarrow -\frac{1}{2} + 1 = \frac{r_2}{2}$$

$\vdots$

$$9 \rightarrow -\frac{1}{9} + 1 = \frac{r_9}{9}$$

$$R = \left\{ \frac{r_1}{1}, \frac{r_2}{2}, \dots, \frac{r_9}{9} \right\}$$

$$\rightarrow \frac{r_1 + \dots + r_9}{9} = \frac{9 \cdot \left(\frac{1+9}{2} \right)}{9} = \frac{r_1 + r_9}{2} = \boxed{5}$$

$$f(n) = \sqrt{an^2 + bn + c}$$

$$D = [r, +\infty) \Rightarrow a = 0 \quad f(n) = \sqrt{bn + c}$$

(V)

$$\begin{cases} rb + c = 0 \\ rb + c = 1 \end{cases}$$

$$\begin{cases} rb + c = 1 \\ b = 1, c = -r \end{cases}$$

$$b = 1, c = -r$$

$$g(n) = \sqrt{n^2 + r}$$

$$-\frac{\Delta}{4a} = -\frac{(0 - 14)}{4} = \frac{7}{2}$$

$$R_g = \sqrt{[r, +\infty)} = \boxed{[r, +\infty)}$$

$$f(n) = \frac{x^r - rn}{n+r} = \frac{x(n^r - r)}{n+r} \xrightarrow{n \neq -r} n(n-r) = n^r - rn$$

(A)

$$D = R - \{a, b\} \quad R = [c, +\infty) - \{\lambda\}$$

$$c = -\frac{\Delta}{4a} = -\frac{r}{r} = -1$$

$$x^r - rn \neq \lambda \Rightarrow n^r - rn - \lambda \neq 0$$

$$(n-r)(n+r) \neq 0$$

$$\begin{cases} n = r \\ n = -r \end{cases} \Rightarrow a, b: r, -r$$

$$f\left(\frac{r-r}{-r}\right) = f(-1) = (-1)^r - r(-1) = \boxed{r}$$

$$g = \sqrt{x^2 + an^2 + bn - \lambda} = \sqrt{(n-r)^2}$$

(9)

$$D = [-4, 12]$$

$$= \sqrt{x^2 - 4n^2 + 12n - \lambda} \quad a = -4, b = 12$$

$$\left. \begin{array}{l} \xrightarrow{-4} -\lambda \\ \xrightarrow{12} 10 \end{array} \right\} \rightarrow R_f = \boxed{[-\lambda, 10]}$$

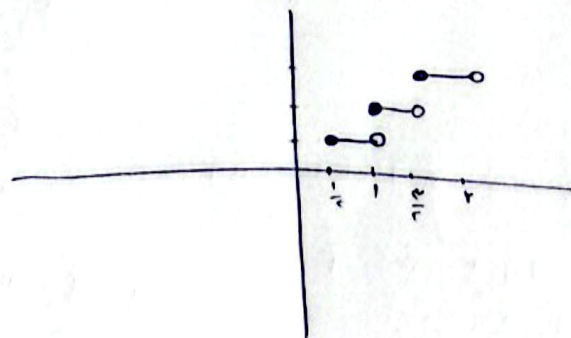
$$y = [n] + \left[n + \frac{1}{2}\right] \quad \left[\frac{1}{2}, r\right)$$

(1)

$$\left[\frac{1}{2}, 1\right) \xrightarrow{[n] \leq 0, [n + \frac{1}{2}] \leq 1} 0 + 1 \leq 1$$

$$\left[1, \frac{3}{2}\right) \xrightarrow{[n] \leq 1, [n + \frac{1}{2}] \leq 2} 1 + 1 \leq 2$$

$$\left[\frac{3}{2}, 2\right) \xrightarrow{[n] \leq 1, [n + \frac{1}{2}] \leq 2} 1 + 1 \leq 2$$



b_0, b_1, b_2