

$$f(x) = 2 \sin^2\left(x + \frac{\pi}{4}\right) - 1$$

$$-1 \leq \sin^2\left(x + \frac{\pi}{4}\right) \leq 1 \xrightarrow{\times 2} -2 \leq 2 \sin^2\left(x + \frac{\pi}{4}\right) \leq 2$$

$$\xrightarrow{-1} -1 \leq f(x) \leq 1$$

$$\Rightarrow R_f = [-1, 1]$$

$$g(x) = 2 \cos^2 x - 4$$

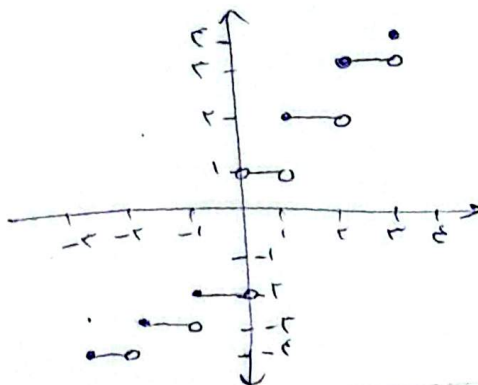
$$-1 \leq \cos^2 x \leq 1 \xrightarrow{\times 2} -2 \leq 2 \cos^2 x \leq 2 \xrightarrow{-4} -6 \leq 2 \cos^2 x - 4 \leq -2$$

$$\Rightarrow R_g = [-6, -2]$$

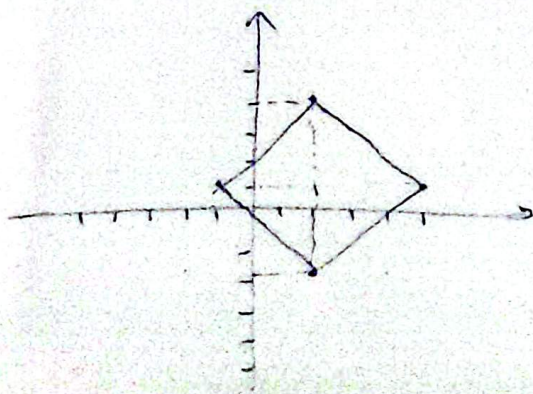
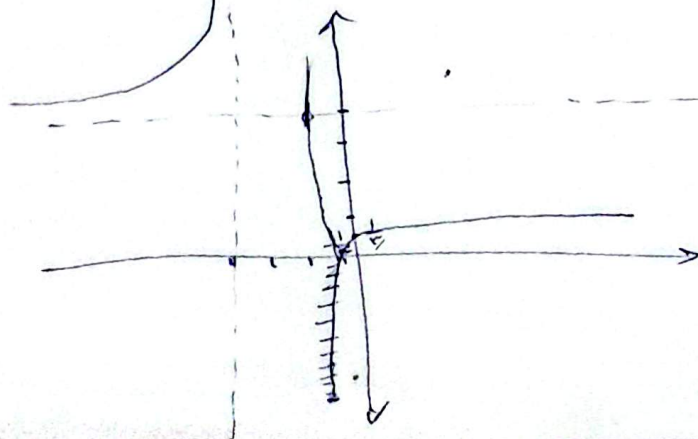
$$y = \frac{x}{|x| + 1} + [x] \quad [-2, 2]$$

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$$\begin{cases} x > 0 \rightarrow 1 + [x] \\ x < 0 \rightarrow [x] - 1 \end{cases}$$



$$y = \left| \frac{x+1}{x+2} \right| \quad -3$$



$$|x-2| + |y-1| = 2$$

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$$\text{مركز } (2, 1)$$

$$y = \begin{cases} x+2 & x \leq -2, y \geq 1 \\ -x+2 & x \geq -2, y \geq 1 \\ x-2 & x \geq 2, y \leq 1 \\ -x+2 & x \leq 2, y \leq 1 \end{cases}$$

در بازه هم در نظر
 الف) $f(u) = u + \frac{1}{u} + r\sqrt{u + \frac{1}{u}}$

کار و مفید

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$$u + \frac{1}{u} \geq r \rightarrow t = u + \frac{1}{u} \rightarrow t \in [r, +\infty)$$

$$y = t + r\sqrt{t} \rightarrow t \geq r \rightarrow y_{\min} = r + r\sqrt{r}$$

$$R_f = [r + r\sqrt{r}, +\infty)$$

ب) $y_2(r - \sin u)(1 + \sin u) = -\sin^2 u + r \sin u + r$

$t = \sin u$
 $-1 \leq \sin u \leq 1$

$\sin u = -1 \rightarrow -1 - r + r = 0 = \min$

$\sin u = 0 \rightarrow 0 + 0 + r = r$

$\sin u = 1 \rightarrow -1 + r + r = 2r - 1 = \max$

$\frac{-b}{2a} = \sin u = \frac{-r}{-r} = 1$

$$R_f = [0, r]$$

$D_f = \{1, 2, 5, \dots, 9\}$ $f(u) = -\frac{1}{r}u + 10$

-4

$u=1 \rightarrow -\frac{1}{r} + 10 = \frac{r9}{r}$

$u=8 \rightarrow \frac{r0}{r}$

$u=2 \rightarrow \frac{r8}{r}$

$u=7 \rightarrow \frac{r1}{r}$

$u=5 \rightarrow \frac{r6}{r}$

$u=6 \rightarrow \frac{r5}{r}$

$u=4 \rightarrow \frac{r4}{r}$

$u=9 \rightarrow \frac{r3}{r}$

$\frac{r9}{r} + \frac{r8}{r} + \frac{r6}{r} + \frac{r4}{r} + \frac{r0}{r} + \frac{r1}{r} + \frac{r5}{r} + \frac{r3}{r} = \textcircled{V0}$

$f(u) = \sqrt{au^2 + bu + c}$ $(r, +\infty) \rightarrow a=0, b>0$

-1

$\sqrt{bu + c} \Rightarrow r b + c = 1 \Rightarrow -rb = c \Rightarrow f(r) = 1$

$c = -rb$

$b=1$

$c=-r$

$y(u) = \sqrt{bu^2 + au + c} = \sqrt{\frac{u^2 + r}{r}}$
 $[r, +\infty)$

$\sqrt{\frac{u^2 + r}{r}} \rightarrow R_g = [r, +\infty)$

$f(x) = \frac{x^2 - 6x}{x + r}$ $x \neq -r$ $\frac{x^2 - 6x}{x + r} = 1$

-1

$\frac{x(x^2 - 6x)}{x + r} = \frac{x(x - r)(x + r)}{(x + r)} = x(x - r) \rightarrow x(x - r) = 1 \rightarrow$

$x^2 - rx - 1 = 0 \rightarrow (x + r)(x - r) = 0 \rightarrow x = r$

$D_f = \mathbb{R} - \{a, b\}$ $\frac{f(x)}{x} = \frac{x^2 - 6x}{x + r}$

$x^2 - 6x = (x - r)^2 - 1$
 $R_f = [r, +\infty)$

$f\left(\frac{u+b}{rc}\right) = \frac{r-r}{-r} = 1$ $f(-1) = (-1)^2 + r = r$

$$y = \sqrt{n^2 + an + b} - 1$$

$$n^2 + an + b - 1 = (n - 1)^2$$

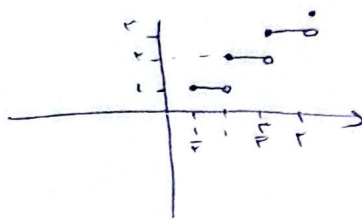
$$(n - 1)^2 = n^2 - 2n + 1 \quad a = -2 \quad b = 1$$

$$y = n - 1 \quad D_f = [-1, 1] \rightarrow R_f = [-1, 1]$$

$$y = [x] + [x + \frac{1}{t}] \quad (\frac{1}{t} > 1)$$

- 1 -

$$\begin{cases} \frac{1}{t} \leq x < 1 \rightarrow 1 \\ 1 \leq x < \frac{t}{t} \rightarrow 2 \\ \frac{t}{t} \leq x < 2 \rightarrow 3 \end{cases}$$



(1, 2, 3)