

الف) $f(x) = \sqrt{\cos^2(x + \frac{\pi}{2})} - 1 \rightarrow R_f = [-1, 1]$

ب) $g(x) = 3\sqrt{\cos^2 x} - 4 \rightarrow R_g = [-1, 1]$

$y = \frac{x}{|x|} + [x]$

$-3 \leq x < -2 \rightarrow y = -1 + (-3) = -4$

$-2 \leq x < -1 \rightarrow y = -1 + (-2) = -3$

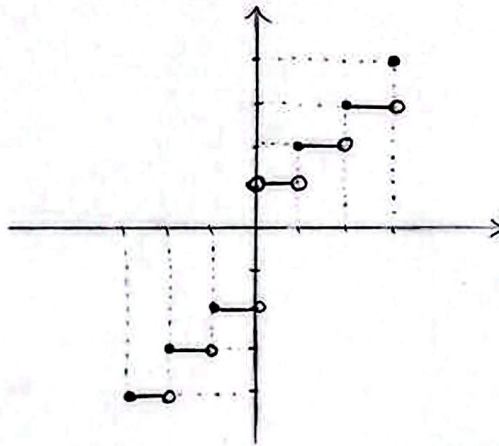
$-1 \leq x < 0 \rightarrow y = -1 + (-1) = -2$

$0 \leq x < 1 \rightarrow y = 1 + 0 = 1$

$1 \leq x < 2 \rightarrow y = 1 + 1 = 2$

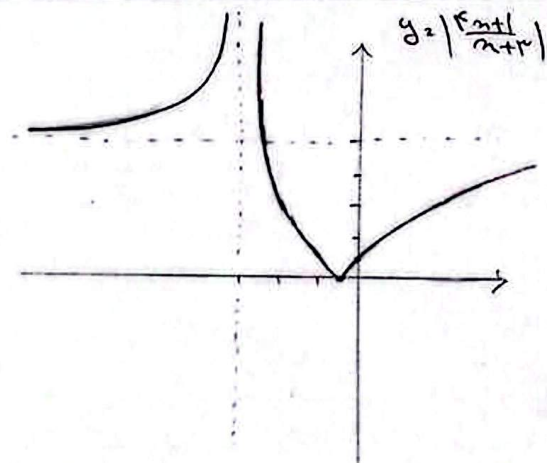
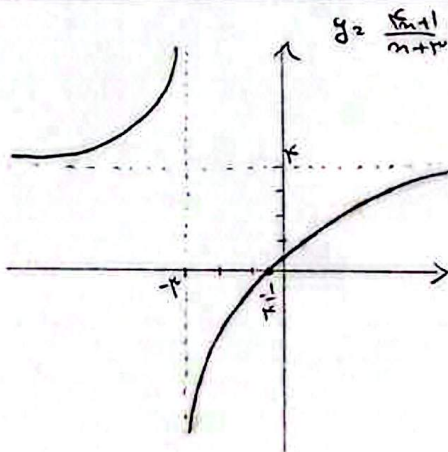
$2 \leq x < 3 \rightarrow y = 1 + 2 = 3$

$x = 3 \rightarrow y = 1 + 3 = 4$



تابع در $x=0$ تعریف نمی شود
زیرا مخرج صفر می شود.

$y = \left| \frac{x+1}{x+3} \right|$



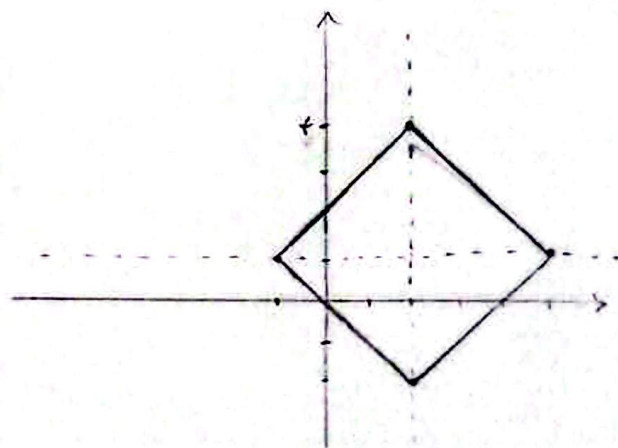
$|x-2| + |y-1| = 3$

$x \geq 2, y \geq 1 \rightarrow x-2+y-1=3 \rightarrow y = -x+6$

$x \geq 2, y < 1 \rightarrow x-2-y+1=3 \rightarrow y = x-4$

$x < 2, y \geq 1 \rightarrow 2-x+y-1=3 \rightarrow y = x+2$

$x < 2, y < 1 \rightarrow 2-x-y+1=3 \rightarrow y = -x$



الف) $f(n) = n + \frac{1}{n} + 2\sqrt{n + \frac{1}{n}}$

تابع $\rightarrow n + \frac{1}{n} > 0$ $\rightarrow n + \frac{1}{n} > 2$

نیم $a > 0$ $\rightarrow a + \frac{1}{a}$

تابع $y = n + \frac{1}{n} + 2\sqrt{n + \frac{1}{n}}$ در $n=1$ مقدار کمترین مقدار آن به ازای $n=1$ است $\rightarrow R_2 = [2, +\infty)$

ب) $y = (3 - \sin n)(1 + \sin n) = -\sin^2 n + 2\sin n + 3 \rightarrow \sin n = 1 \rightarrow y = 4 \rightarrow R = [2, 4]$

$f(n) = \frac{1}{n} + 10$

$n=1 \rightarrow f(1) = \frac{1}{1} + 10 = 11$

$n=2 \rightarrow f(2) = \frac{1}{2} + 10 = 10.5$

$n=3 \rightarrow f(3) = \frac{1}{3} + 10 = 10.33$

$n=4 \rightarrow f(4) = \frac{1}{4} + 10 = 10.25$

$n=5 \rightarrow f(5) = \frac{1}{5} + 10 = 10.2$

$n=6 \rightarrow f(6) = \frac{1}{6} + 10 = 10.16$

$n=7 \rightarrow f(7) = \frac{1}{7} + 10 = 10.14$

$n=8 \rightarrow f(8) = \frac{1}{8} + 10 = 10.125$

$n=9 \rightarrow f(9) = \frac{1}{9} + 10 = 10.11$

مقدار $f(n) = \frac{1}{n} + 10 = \frac{1}{1} + 10 = 11$

$f(n) = \sqrt{an^2 + bn + c}$

$D_f = [1, +\infty) \rightarrow [a > 0]$

$bn + c > 0 \rightarrow n > -\frac{c}{b} \rightarrow -\frac{c}{b} > 2 \rightarrow c < -2b$

$f(2) = 1 \rightarrow 4b + c = 1 \rightarrow [b < 1] [c < -2]$

$\rightarrow g(n) = \sqrt{n^2 + 4} \rightarrow R_g = [2, +\infty)$

$f(n) = \frac{n^3 - 4n}{n+2} = \frac{n(n-2)(n+2)}{(n+2)} = n(n-2) = n^2 - 2n \rightarrow R_f = [1, +\infty)$

$D_f = \mathbb{R} - \{a, b\}$ $\rightarrow$ چون نقطه $n=2$ صفر می شود $a=b=2$ $\rightarrow$ تعریف نشده است $\rightarrow$ در $n=2$ مقدار از $n=1$ کمتر است

$f\left(\frac{a+b}{2}\right) = f\left(\frac{-2+(-2)}{2(-1)}\right) = f(2) = 0$

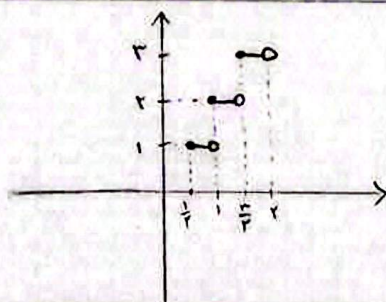
$g = \sqrt[3]{n^3 + an^2 + bn - 1}$

$(n-2)^3 = n^3 - 6n^2 + 12n - 8 \rightarrow a = -6, b = 12$

$D_f = [-4, 12] \rightarrow R_f = [-1, 10]$

چون تابع خطی است متوجه می شویم زیر رادیکال یک مربع کامل است

$g = \lfloor n \rfloor + \lfloor n + \frac{1}{n} \rfloor$
 $\frac{1}{n} < 1 \rightarrow g = 0 + 1 = 1$
 $1 < n < \frac{3}{2} \rightarrow g = 1 + 1 = 2$
 $\frac{3}{2} < n < 2 \rightarrow g = 1 + 2 = 3$



از 3 باره خط تشکیل شده است