

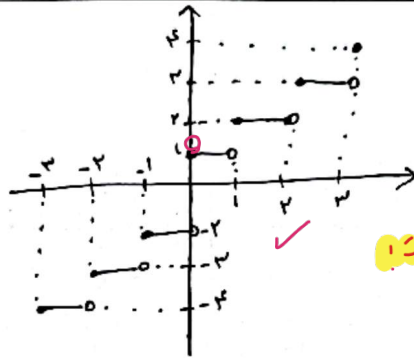
$$\text{الف) } 0 \leq \sin^2(x + \frac{\pi}{4}) \leq 1 \Rightarrow 0 \leq 2 \sin^2(x + \frac{\pi}{4}) \leq 2 \Rightarrow -1 \leq 2 \sin(x + \frac{\pi}{4}) - 1 \leq 1$$

$$\Rightarrow -1 \leq f(x) \leq 1 \Rightarrow R_f = [-1, 1] \checkmark$$

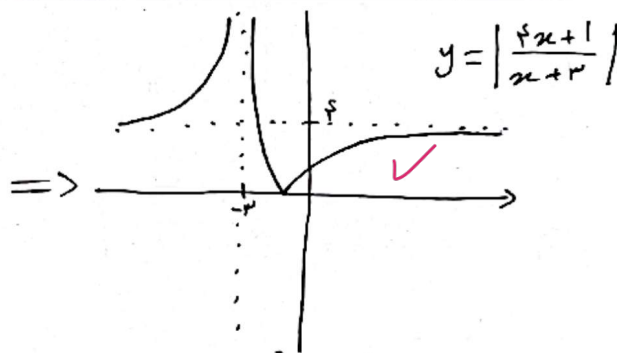
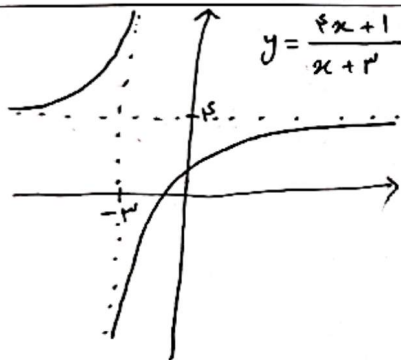
$$\text{ب) } -1 \leq \cos x \leq 1 \Rightarrow -1 \leq \cos^3 x \leq 1 \Rightarrow -3 \leq 3 \cos^3 x \leq 3$$

$$\Rightarrow -\sqrt{3} \leq 3 \cos^3 x - 4 \leq -1 \Rightarrow -\sqrt{3} \leq g(x) \leq -1 \Rightarrow R_g = [-\sqrt{3}, -1] \checkmark$$

$$y = \begin{cases} -4 & x \in [-3, -2) \\ -3 & x \in [-2, -1) \\ -2 & x \in [-1, 0) \\ 1 & x \in [0, 1) \\ 2 & x \in [1, 2) \\ 3 & x \in [2, 3) \\ 4 & x = 3 \end{cases} \Rightarrow$$

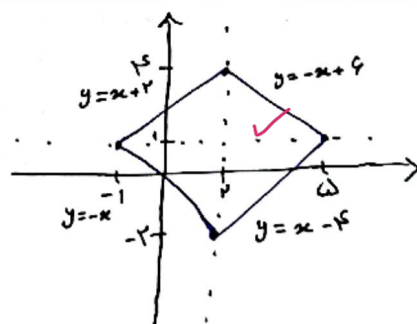


صفر تعریف شده است



$$\begin{cases} x - y + 1 = 3 \Rightarrow y = -x + 4 : x \geq 2, y \geq 1 \\ x - y + 1 = 3 \Rightarrow y = x - 4 : x \geq 2, y < 1 \\ 2 - x + y - 1 = 3 \Rightarrow y = x + 2 : x < 2, y \geq 1 \\ 2 - x - y + 1 = 3 \Rightarrow y = -x : x < 2, y < 1 \end{cases}$$

دو به دو محل تقاطع خطوط مجاور را پیدا کرده و به نقاط (۱، ۵)، (۲، ۲)، (۲، ۴) و (۳، ۳) رسم می‌کنیم.



$$\text{الف) } x + \frac{1}{x} \geq 2 \wedge x + \frac{1}{x} \leq -2, \quad x + \frac{1}{x} \geq 0 \Rightarrow x + \frac{1}{x} \geq 2$$

$$x + \frac{1}{x} = t \Rightarrow f(x) = t + 2\sqrt{t} \Rightarrow \begin{cases} \sqrt{t} = \sqrt{t} \\ \sqrt{t} = \frac{b}{\sqrt{a}} = 1 \Rightarrow \sqrt{t} = 1 \end{cases} \Rightarrow f(x) = 2 + 2\sqrt{t} \Rightarrow R_f = [2 + 2\sqrt{t}, +\infty) \checkmark$$

$$\text{ب) } y = -\sin^2 x + 2 \sin x + 3 \quad -1 \leq \sin x \leq 1 \Rightarrow \begin{cases} \sin x = -1 \Rightarrow y = 0 \\ \sin x = \frac{b}{a} = 1 = \max \sin x \Rightarrow y = 4 \end{cases} \Rightarrow R = [0, 4] \checkmark$$

$$\begin{aligned} x=1 &\rightarrow 10 - \frac{1}{r} \\ x=2 &\rightarrow 10 - \frac{2}{r} \\ x=3 &\rightarrow 10 - 1 = 9 \\ x=4 &\rightarrow 10 - \frac{4}{r} \\ x=5 &\rightarrow 10 - \frac{5}{r} \\ x=6 &\rightarrow 10 - \frac{6}{r} \\ x=7 &\rightarrow 10 - \frac{7}{r} \\ x=8 &\rightarrow 10 - \frac{8}{r} \\ x=9 &\rightarrow 10 - \frac{9}{r} = 9 \end{aligned}$$

$$\begin{aligned} 10 - \frac{1}{r} + 10 - \frac{2}{r} + 9 + 10 - \frac{4}{r} + 10 - \frac{5}{r} \\ + 10 - \frac{6}{r} + 10 - \frac{7}{r} + 9 = 90 + 24 - \frac{1+2+4+5+6+7+8}{r} \\ = 114 - \frac{35}{r} = 114 - 9 = 105 \end{aligned}$$

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$$D_f = [2, +\infty) \Rightarrow \text{تابع خطی است } ax^2 + bx + c \Rightarrow a=0$$

$$\hookrightarrow f(2) = 0 \Rightarrow 2b + c = 0$$

$$f(3) = 1 \Rightarrow 3b + c = 1 \Rightarrow b + \frac{2b+c}{2} = 1 \Rightarrow b = 1 \Rightarrow c = -2$$

$$\Rightarrow g(x) = \sqrt{(1)x^2 + (0)x + 4} = \sqrt{x^2 + 4} \Rightarrow x^2 + 4 \geq 4 \Rightarrow \sqrt{x^2 + 4} \geq 2 \Rightarrow R = [2, +\infty)$$

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$$D_f = R - \{-2\} \Rightarrow \begin{cases} a=0, b=-2 \\ a=-2, b=0 \end{cases} \Rightarrow a+b=-2$$

$$f(x) = \frac{x(x-2)(x+2)}{x^2+4} = x^2 - 4 \Rightarrow R_f = \left[\frac{-4}{f(x)}, +\infty \right) \Rightarrow c = \frac{-4}{f(x)} = \frac{f(x)c - b^2}{f(x)} = \frac{-4}{f(x)} = -1$$

$$f\left(\frac{a+b}{2}\right) = f\left(\frac{-2}{-2}\right) = f(1) = \frac{(1)^2 - 4(1)}{1+2} = \frac{-3}{3} = -1$$

صفحه آخر...

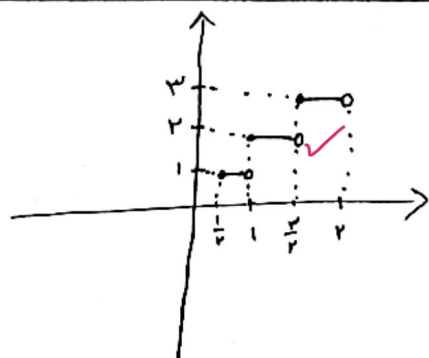
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$$\text{تابع خطی است} \Rightarrow y = \sqrt[3]{(x-2)^3} = \sqrt[3]{x^3 - 6x^2 + 12x - 8} \Rightarrow \begin{cases} a=-6 \\ b=12 \end{cases}$$

$$y = x-2 \xrightarrow{x \in [-6, 12]} -6 \leq x \leq 12 \Rightarrow -1 \leq x-2 \leq 10 \Rightarrow R = [-1, 10]$$

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$$\begin{aligned} 0+1 &= 1 : \left[\frac{1}{2}, 1\right) \\ 1+1 &= 2 : \left[1, \frac{3}{2}\right) \\ 1+2 &= 3 : \left[\frac{3}{2}, 2\right) \end{aligned}$$



(خط 1/2, 3)

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$$f(x) = \frac{x(x-2)(x+2)}{x+2} = x^2 - 2x$$

$$x^2 - 2x \neq 8 \rightarrow \begin{cases} x \neq -2 \\ x \neq 4 \end{cases}$$

$y = 8$ در برد تابع نیست یعنی:

پس دامنه تابع است: $D_f = \mathbb{R} - \{-2, 4\}$

$$f(x) = x^2 - 2x \rightarrow \lim_{x \rightarrow -1} \left| \frac{1}{-1} \right| \Rightarrow R_f = [-1, +\infty) - \{8\} \quad \text{حالا بیایم سراغ برد!}$$

$$f\left(\frac{a+b}{2c}\right) = f\left(\frac{2}{-2}\right) = f(-1) \rightarrow f(-1) = \textcircled{3}$$