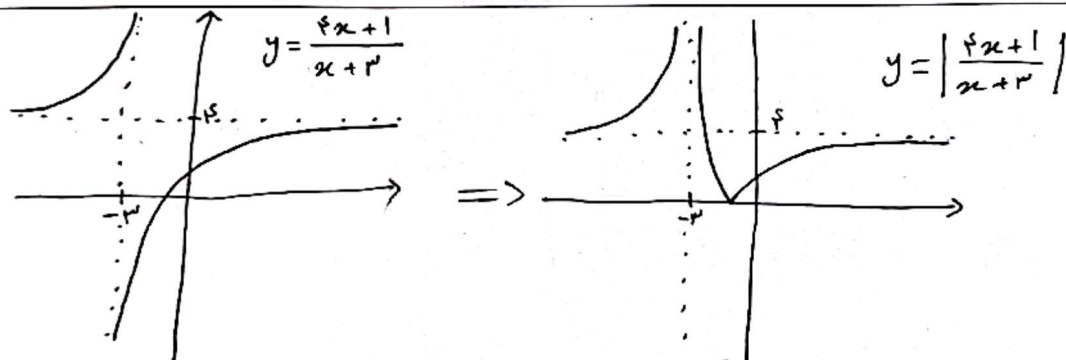
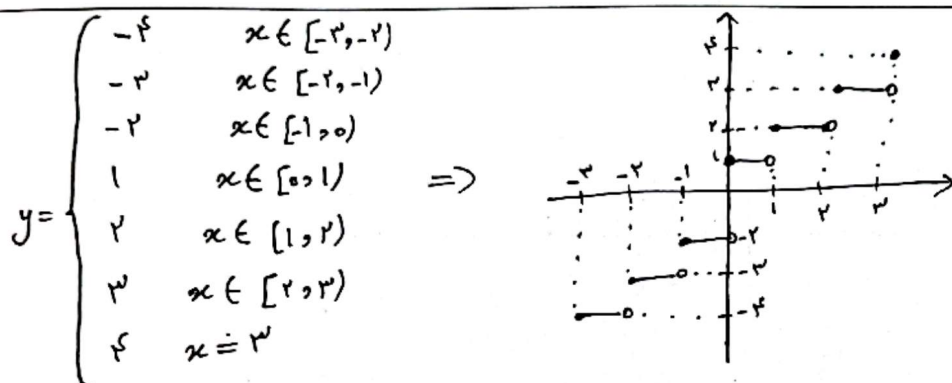
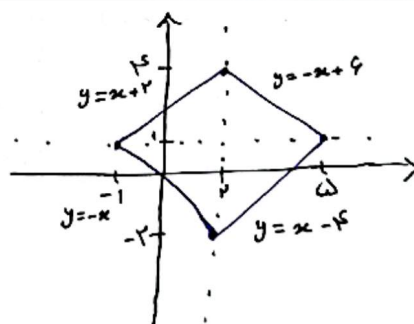


الف) $0 \leq \sin^2(x + \frac{\pi}{4}) \leq 1 \Rightarrow 0 \leq 2 \sin^2(x + \frac{\pi}{4}) \leq 2 \Rightarrow -1 \leq 2 \sin^2(x + \frac{\pi}{4}) - 1 \leq 1$
 $\Rightarrow -1 \leq f(x) \leq 1 \Rightarrow R_f = [-1, 1]$

ب) $-1 \leq \cos x \leq 1 \Rightarrow -1 \leq \cos^3 x \leq 1 \Rightarrow -3 \leq 3 \cos^3 x \leq 3$
 $\Rightarrow -4 \leq 3 \cos^3 x - 1 \leq -1 \Rightarrow -4 \leq g(x) \leq -1 \Rightarrow R_g = [-4, -1]$



$\begin{cases} x-2+y-1=3 \Rightarrow y=-x+6 : x \geq 2, y \geq 1 \\ x-2-y+1=3 \Rightarrow y=x-4 : x \geq 2, y < 1 \\ 2-x+y-1=3 \Rightarrow y=x+2 : x < 2, y \geq 1 \\ 2-x-y+1=3 \Rightarrow y=-x : x < 2, y < 1 \end{cases}$
 دو به دو محل تقاطع خطوط مجاور را پیدا کرده و به نقاط (۱، ۱)، (۲، ۲)، (۲، ۴) و (۴، ۲) رسم می‌کنیم.



الف) $x + \frac{1}{x} \geq 2 \wedge x + \frac{1}{x} \leq -2, \quad x + \frac{1}{x} \geq 0 \Rightarrow x + \frac{1}{x} \geq 2$
 $x + \frac{1}{x} = t \Rightarrow f(x) = t + 2\sqrt{t} \Rightarrow \begin{cases} \sqrt{t} = \sqrt{t} \\ \sqrt{t} = \frac{b}{\sqrt{a}} = -1 \text{ و } \sqrt{t} = 1 \end{cases} \Rightarrow f(x) = 2 + 2\sqrt{t} \Rightarrow R_f = [2 + 2\sqrt{t}, +\infty)$
 ب) $y = -\sin^2 x + 2 \sin x + 3$
 $-1 \leq \sin x \leq 1 \Rightarrow \begin{cases} \sin x = -1 \Rightarrow y = 0 \\ \sin x = \frac{b}{a} = 1 = \max \sin x \Rightarrow y = 4 \end{cases} \Rightarrow R = [0, 4]$

$$\begin{aligned} x=1 &\rightarrow 10 - \frac{1}{r} \\ x=2 &\rightarrow 10 - \frac{2}{r} \\ x=3 &\rightarrow 10 - 1 = 9 \\ x=4 &\rightarrow 10 - \frac{4}{r} \\ x=5 &\rightarrow 10 - \frac{5}{r} \\ x=6 &\rightarrow 10 - \frac{6}{r} = 8 \\ x=7 &\rightarrow 10 - \frac{7}{r} \\ x=8 &\rightarrow 10 - \frac{8}{r} \\ x=9 &\rightarrow 10 - \frac{9}{r} = 7 \end{aligned}$$

$$\begin{aligned} 10 - \frac{1}{r} + 10 - \frac{2}{r} + 9 + 10 - \frac{4}{r} + 10 - \frac{5}{r} \\ + 8 + 10 - \frac{6}{r} + 10 - \frac{7}{r} + 7 = 90 + 24 - \frac{1+2+4+5+6+7+8}{r} \\ = 114 - \frac{35}{r} = 114 - 9 = 105 \end{aligned}$$

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$$D_f = [2, +\infty) \Rightarrow \text{تابع خطی است } ax^2 + bx + c \Rightarrow a=0$$

$$\Rightarrow f(2)=0 \Rightarrow 2b+c=0$$

$$f(3)=1 \Rightarrow 3b+c=1 \Rightarrow b + \frac{2b+c}{2} = 1 \Rightarrow b=1 \Rightarrow c=-2$$

$$\Rightarrow g(x) = \sqrt{(1)x^2 + (0)x + 4} = \sqrt{x^2 + 4} \Rightarrow x^2 + 4 \geq 4 \Rightarrow \sqrt{x^2 + 4} \geq 2 \Rightarrow R = [2, +\infty)$$

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$$D_f = R - \{-2\} \Rightarrow \begin{cases} a=0, b=-2 \\ a=-2, b=0 \end{cases} \Rightarrow a+b=-2$$

$$f(x) = \frac{x(x-2)(x+2)}{x^2+4} = x^2 - 2x \Rightarrow R_f = \left[\frac{-4}{4}, +\infty \right) \Rightarrow c = \frac{-4}{4} = \frac{f(a)-b^2}{4a} = \frac{-4}{4} = -1$$

8

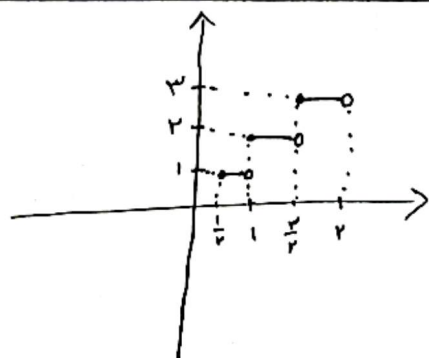
$$f\left(\frac{a+b}{2}\right) = f\left(\frac{-2}{-2}\right) = f(1) = \frac{(1)^2 - 4(1)}{1+2} = \frac{-3}{3} = -1$$

$$\text{تابع خطی است} \Rightarrow y = \sqrt[3]{(x-2)^3} = \sqrt[3]{x^3 - 6x^2 + 12x - 8} \Rightarrow \begin{cases} a=-6 \\ b=12 \end{cases}$$

$$y = x-2 \xrightarrow{x \in [-6, 12]} -6 \leq x \leq 12 \Rightarrow -8 \leq x-2 \leq 10 \Rightarrow R = [-8, 10]$$

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$$\begin{aligned} 0+1 &= 1 : \left[\frac{1}{2}, 1\right) \\ 1+1 &= 2 : \left[1, \frac{3}{2}\right) \\ 1+2 &= 3 : \left[\frac{3}{2}, 2\right) \end{aligned}$$



(تابع پله‌ای)

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