

①

الف) $2\sin^2(x + \frac{\pi}{3}) - 1 \Rightarrow 0 \leq \sin^2(x + \frac{\pi}{3}) \leq 1 \xrightarrow{\times 2} 0 \leq 2\sin^2(x + \frac{\pi}{3}) \leq 2 \xrightarrow{-1} -1 \leq 2\sin^2(x + \frac{\pi}{3}) - 1 \leq 1$

$-1 \leq 2\sin^2(x + \frac{\pi}{3}) - 1 \leq 1$

ب) $3\cos^3 x - 4 \Rightarrow -1 \leq \cos^3 x \leq 1 \xrightarrow{\times 3} -3 \leq 3\cos^3 x \leq 3 \xrightarrow{-4} -7 \leq 3\cos^3 x - 4 \leq -1$

$[-3, 2] \rightarrow y = -1 + (-3) = -4$

$[-2, -1] \rightarrow y = -1 + (-2) = -3$

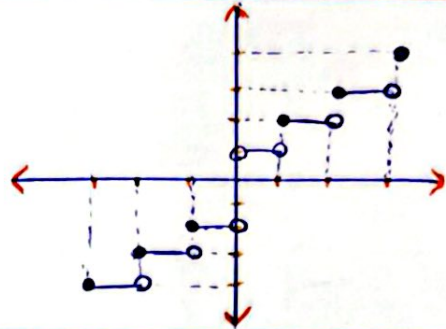
$[-1, 0] \rightarrow y = -1 + (-1) = -2$

$(0, 1] \rightarrow y = 1 + 0 = 1$

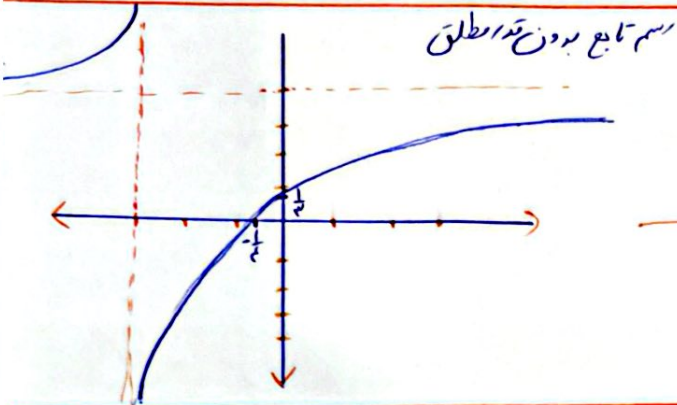
$[1, 2] \rightarrow y = 1 + 1 = 2$

$[2, 3] \rightarrow y = 1 + 2 = 3$

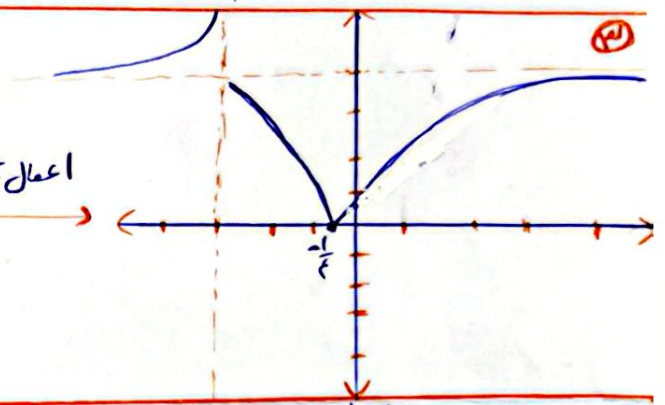
$[3] \rightarrow y = 1 + 3 = 4$



②



اعمال قید و ملق



③

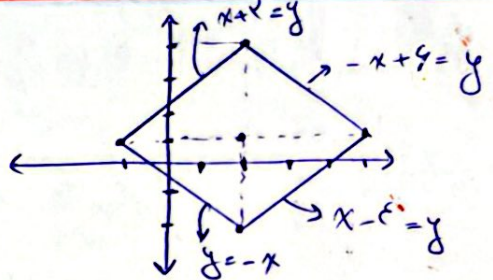
$|x-2| + |y-1| = 3$

$x > 2, y > 1 \rightarrow x-2+y-1=3 \Rightarrow -x+y=y$

$x > 2, y < 1 \rightarrow x-2-y+1=3 \Rightarrow x-4=y$

$x < 2, y > 1 \rightarrow -x+2+y-1=3 \Rightarrow x+y=y$

$x < 2, y < 1 \rightarrow -x+2-y+1=3 \Rightarrow y=-x$



④

الف) $f(x) = x + \frac{1}{x} + 2\sqrt{x + \frac{1}{x}} \xrightarrow{x + \frac{1}{x} \geq 0} a + \frac{1}{a} \geq 0 \xrightarrow{\times a} \{2, +\infty\}$

$x + \frac{1}{x} = t \rightarrow t + 2\sqrt{t} \xrightarrow{R_t = \{2, +\infty\}} \min y = 2 \Rightarrow 2 + 2\sqrt{2} \rightarrow [2 + 2\sqrt{2}, +\infty) = R_f$

⑤

ب) $y = (e^{\sin x})(1 + \sin x) \Rightarrow y = -\sin^2 x + 2\sin x + 2$

$\sin x \begin{cases} -1 \\ 1 \end{cases} \Rightarrow \sin x \begin{cases} 1 \\ -1 \end{cases} \rightarrow \begin{cases} -(1)^2 + 2(1) + 2 = -1 + 2 + 2 = 3 \\ -(-1)^2 + 2(-1) + 2 = -1 - 2 + 2 = -1 \end{cases}$

$R_f = [0, 3]$

$\frac{-b}{2a} = \frac{2}{-2} = -1$

$$f(x) = -\frac{1}{x}x + 10 \xrightarrow{\text{دانه}} D_f = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

$$\xrightarrow{\text{مقادیر مثبت و منفی}} R_f = \left\{ \frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9} \right\}$$

$$\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} = \frac{223}{252} = \frac{75}{84}$$

مجموع برد $\Rightarrow$

$$f(x) = \sqrt{a(x-2)^2}$$

چون دامنه $\{2+\infty\}$ عبارت ریشه مضاعف دارد $\Rightarrow$

$$f(2)=1 \rightarrow \sqrt{a}=1 \Rightarrow \boxed{a=1} \rightarrow f(x) = x^2 - 4x + 4 \Rightarrow \boxed{b=-4}, \boxed{c=4}$$

$$g(x) = \sqrt{-4x^2 + x - 1} \xrightarrow{\Delta = b^2 - 4ac = -4} -4x^2 + x - 1 \rightarrow \boxed{\Delta < 0} \Rightarrow \text{است بی‌عبارت} \Rightarrow \text{چون ضریب } x^2 \text{ منفی است و دایره را در کمال تعریف نمی‌شود.}$$

$$\boxed{R_g = \emptyset}$$

$$f(x) = \frac{x^2 - 4x}{x+2} = \frac{x(x-2)(x+2)}{(x+2)} \Rightarrow \boxed{x \neq -2}$$

$$1 = \frac{x(x-2)(x+2)}{(x+2)} \Rightarrow x^2 - 4x = 1 \Rightarrow (x-4)(x+1) = 0 \Rightarrow x = \begin{matrix} 4 \\ -1 \end{matrix} \Rightarrow \boxed{ab = 4, -2}$$

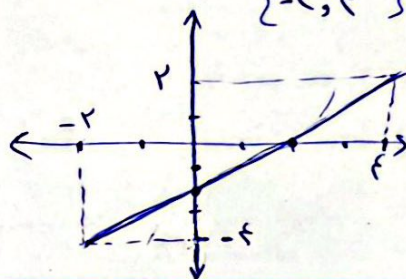
$$\text{بر} \Rightarrow \frac{x(x-2)(x+2)}{(x+2)} \Rightarrow x^2 - 4x \Rightarrow (x-1)^2 - 1 \Rightarrow \text{دایره} \Rightarrow \boxed{(-1, +\infty) - \{1\}}$$

$$f\left(\frac{a+b}{2}\right) \Rightarrow \boxed{f(-1) = 3}$$

$$y = \sqrt[3]{x^3 + ax^2 + bx - 1} \Rightarrow y = \sqrt[3]{(x-2)^3} \Rightarrow y = \sqrt[3]{x^3 - 6x^2 + 12x - 8} \Rightarrow \boxed{a=-6}, \boxed{b=12}$$

دامنه $\in [-2, 4]$

$$y = \sqrt[3]{(x-2)^3} \Rightarrow \boxed{y = x-2} \begin{matrix} x=2 \rightarrow 0 \\ x=4 \rightarrow 2 \end{matrix}$$



$$y = \left\lfloor x \right\rfloor + \left\lfloor x + \frac{1}{2} \right\rfloor \xrightarrow{\text{دانه}} \left\lfloor \frac{1}{2}, 2 \right\rangle$$

$$\left\lfloor \frac{1}{2}, 1 \right\rangle \rightarrow 0 + 1 = 1$$

$$\left\lfloor 1, \frac{3}{2} \right\rangle \rightarrow 1 + 1 = 2$$

$$\left\lfloor \frac{3}{2}, 2 \right\rangle \rightarrow 1 + 2 = 3$$

