

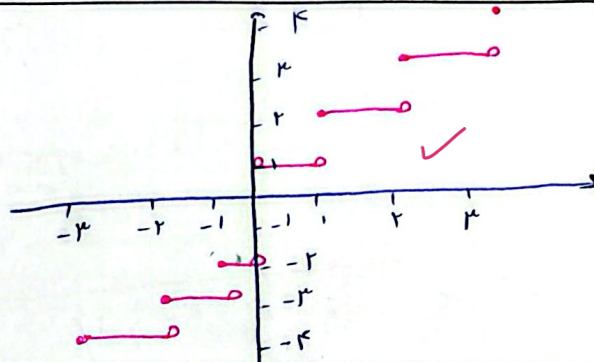
الف) $f(x) = 2\sin^2(x + \frac{\pi}{4}) - 1 \rightarrow -1 \leq \sin x \leq 1 \rightarrow 0 \leq \sin^2 x \leq 1$
 $\rightarrow 0 \leq 2\sin^2 x \leq 2 \rightarrow -1 \leq 2\sin^2 x - 1 \leq 1 \rightarrow R_f = [-1, 1]$

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ب) $g(x) = 3\cos^3 x - 4 \rightarrow -1 \leq \cos x \leq 1 \rightarrow -1 \leq \cos^3 x \leq 1$
 $-3 \leq 3\cos^3 x \leq 3 \rightarrow -7 \leq 3\cos^3 x - 4 \leq -1 \rightarrow R_g = [-7, -1]$

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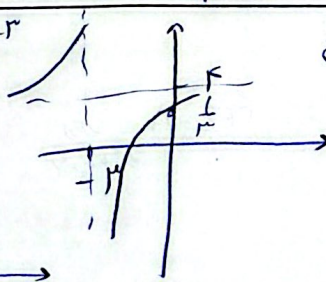
$-3 \leq x < -2 \rightarrow y = -1 - 3 = -4$
 $-2 \leq x < -1 \rightarrow y = -1 - 2 = -3$
 $-1 \leq x < 0 \rightarrow y = -1 - 1 = -2$
 $0 \leq x < 1 \rightarrow y = 1 + 0 = 1$
 $1 \leq x < 2 \rightarrow y = 1 + 1 = 2$
 $2 \leq x < 3 \rightarrow y = 1 + 2 = 3$
 $x = 3 \rightarrow y = 3$



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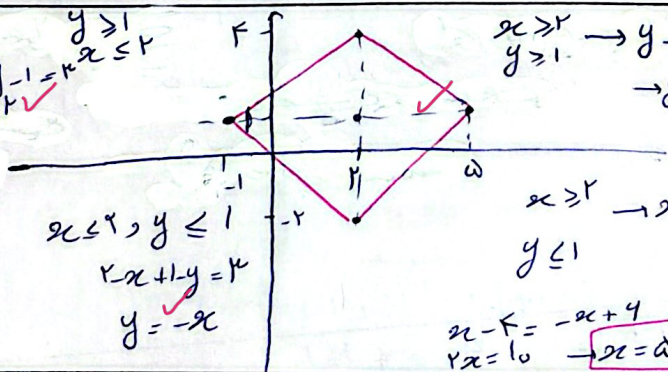
$y = \left| \frac{x+1}{x+3} \right|$
 گانج: $x+3=0 \rightarrow x=-3$
 گانج: $y=1$
 $\rightarrow y = \left| \frac{x+1}{x+3} \right|$



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$x \geq 2, y \geq 1 \rightarrow y - 1 + x - 2 = 3 \rightarrow y = -x + 4$
 $x \geq 2, y \leq 1 \rightarrow x - 2 + 1 - y = 3 \rightarrow y = x - 4$
 $x \leq 2, y \geq 1 \rightarrow x - 2 + y - 1 = 3 \rightarrow y = -x + 6$
 $x \leq 2, y \leq 1 \rightarrow x - 2 + y - 1 = 3 \rightarrow y = -x + 6$
 $x = -1, y = 1$
 $x = -1, y = -1$
 $x = 1, y = 1$
 $x = 1, y = -1$



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الف) $f(x) = x + \frac{1}{x} + 2\sqrt{x + \frac{1}{x}}$
 $x + \frac{1}{x} \geq 0 \rightarrow \frac{x^2 + 1}{x} \geq 0 \rightarrow x > 0$
 $\rightarrow \min f = \min (x + \frac{1}{x}) = 2 \rightarrow R_f = [2 + 2\sqrt{2}, +\infty)$

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ب) $y = 3 + 3\sin x - \sin x - \sin^2 x = -\sin^2 x + 2\sin x + 3$
 $R_y = [2, 4]$

$$f(x) = -\frac{1}{3}x + 1 \rightarrow \begin{cases} x=1 \rightarrow y = \frac{2}{3} \\ x=2 \rightarrow y = \frac{1}{3} \\ \vdots \\ x=9 \rightarrow y = 0 \end{cases}$$

$$\frac{29 + 28 + \dots + 21}{3} = \frac{220}{3} = 73.\bar{3}$$

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تقریباً $\rightarrow$ مقابله $\rightarrow a=0$ $f(1)=0 \rightarrow \sqrt{2b+c}=0 \rightarrow 2b+c=0$

$$f(3)=1 \rightarrow \sqrt{9b+c}=1 \rightarrow 9b+c=1$$

$$\begin{cases} 2b+c=1 \\ 9b+c=0 \end{cases} \rightarrow \begin{matrix} b=1 \\ c=-2 \end{matrix}$$

$$g(x) = \sqrt{x^2 + 1} \rightarrow \min g(x) = 1 \rightarrow g(x) = 1$$

$$R_g = [1, +\infty)$$

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$$f(x) = \frac{x^2 - 4x}{x+2} \Rightarrow D_f = \mathbb{R} - \{-2\}$$

$$f(x) = \frac{x(x-4)}{x+2} = \frac{x(x+2)(x-2)}{x+2} = x(x-2)$$

$$f(x) = x^2 - 2x$$

$$f(-2) = 4 + 8 = 12$$

$$C=1, a=b=-2, f\left(-\frac{1}{2}\right) = f(2) =$$

$$\frac{1-1}{2} = 0$$

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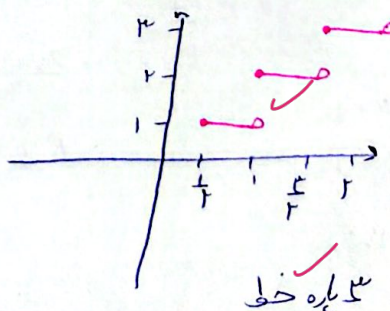
معادله را حل کنید $\rightarrow x^2 + ax + b = 1 \Rightarrow x^2 - 4x + 12 = 1 \Rightarrow x^2 - 4x + 11 = 0$

$$\rightarrow a = -4, b = 11 \rightarrow \Delta = 16 - 44 = -28 < 0$$

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$$\begin{aligned} \frac{1}{3} \leq x < 1 &\rightarrow [x] = 0 \rightarrow y = 0 + 1 = 1 \\ 1 \leq x < \frac{4}{3} &\rightarrow [x] = 1 \rightarrow y = 1 + 1 = 2 \\ \frac{4}{3} \leq x < 2 &\rightarrow [x] = 1 \rightarrow y = 1 + 2 = 3 \end{aligned}$$



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$$f(x) = \frac{x(x-2)(x+2)}{x+2} = x^2 - 2x$$

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$$x^2 - 2x \neq 8 \rightarrow \begin{cases} x \neq -2 \\ x \neq 4 \end{cases}$$

$y=8$ درجه دوم نیست یعنی:

پس دامنه برابر است با: $D_f = \mathbb{R} - \{-2, 4\}$

$$f(x) = x^2 - 2x \rightarrow \lim_{x \rightarrow -1} \left| \frac{1}{-1} \right| \Rightarrow R_f = [-1, +\infty) - \{8\} \quad \text{حالا بیایم سراغ برد!}$$

$$f\left(\frac{a+b}{2c}\right) = f\left(\frac{2}{-2}\right) = f(-1) \rightarrow f(-1) = \textcircled{3}$$