

الف) $f(x) = 2\sin^2(x + \frac{\pi}{4}) - 1 \rightarrow 0 \leq \sin^2(x + \frac{\pi}{4}) \leq 1 \rightarrow 0 \leq 2\sin^2(x + \frac{\pi}{4}) \leq 2 \rightarrow -1 \leq 2\sin^2(x + \frac{\pi}{4}) - 1 \leq 1$

$\Rightarrow R_f = [-1, 1]$

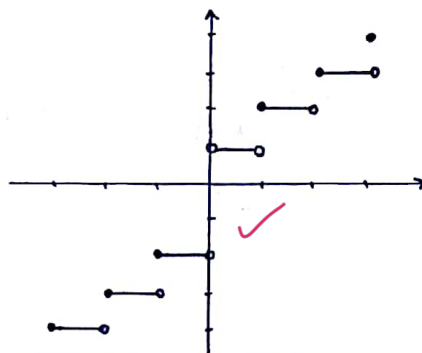
ب) $g(x) = 4\cos^2 x - 4 \rightarrow -1 \leq \cos^2 x \leq 1 \rightarrow -4 \leq 4\cos^2 x \leq 4 \rightarrow -4 \leq 4\cos^2 x - 4 \leq 0$

$\Rightarrow R_g = [-4, 0]$

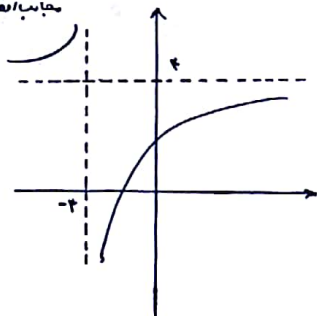
$y = \frac{x}{|x|} + [x]$

$y = \begin{cases} [x] + 1 & x > 0 \\ [x] - 1 & x < 0 \end{cases}$

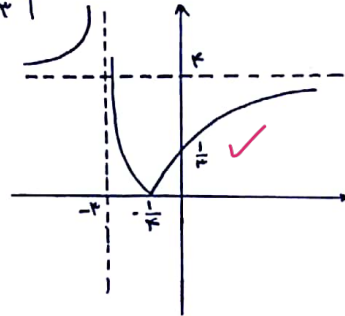
$0 < x < 1 \rightarrow 1$
 $1 \leq x < 2 \rightarrow 2$
 $2 \leq x < 3 \rightarrow 3$
 $x \geq 3 \rightarrow 4$
 $-1 \leq x < 0 \rightarrow -1$
 $-2 \leq x < -1 \rightarrow -2$
 $-3 \leq x < -2 \rightarrow -3$



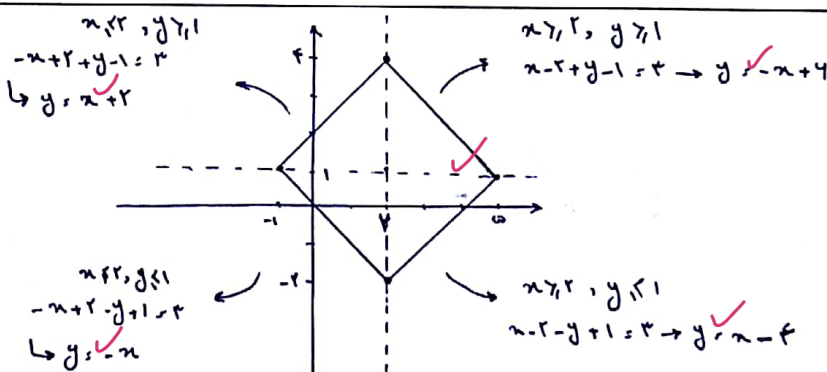
$y = \frac{x+1}{x+3} \rightarrow \begin{cases} -3: \text{میان بر} \\ 3: \text{میان بر} \end{cases}$



$y = \left| \frac{x+1}{x+3} \right|$



$|x-2| + |y-1| = 3$



الف) $f(x) = x + \frac{1}{x} + 2\sqrt{x + \frac{1}{x}}$

$\sqrt{x + \frac{1}{x}} = t \rightarrow \begin{cases} \min t = \sqrt{2} \rightarrow f(x) = 2 + 2\sqrt{2} \\ \max t = +\infty \rightarrow f(x) = +\infty \\ -\frac{b}{2a} = -1 \text{ غرضه} \end{cases} \rightarrow R_f = [2 + 2\sqrt{2}, +\infty)$

ب) $y = (2 - \sin x)(1 + \sin x) = 2 + 2\sin x - \sin x - \sin^2 x = -\sin^2 x + 2\sin x + 2$

$\sin x = t \rightarrow \begin{cases} \min t = -1 \rightarrow y = 0 \\ \max t = 1 \rightarrow y = 4 \\ -\frac{b}{2a} = 1 \rightarrow y = 4 \end{cases} \rightarrow R_f = [0, 4]$

$$f(n) = -\frac{1}{n}n + 10 \rightarrow D_f: \{1, 2, \dots, 9\}$$

$$f(1) + f(2) + f(3) + \dots + f(9) = \left(-\frac{1}{1}(1) + 10\right) + \left(-\frac{1}{2}(2) + 10\right) + \left(-\frac{1}{3}(3) + 10\right) + \dots + \left(-\frac{1}{9}(9) + 10\right) =$$

$$-\frac{1}{1}(1+2+3+\dots+9) + \underbrace{(9 \times 10)}_{\frac{9 \times 10}{2} = 45} = -10 + 45 = 35 \quad \text{✓}$$

2
6

$$f(n) = \sqrt{an^2 + bn + c} \quad D_f: [r, +\infty)$$

حیث دالة محدبة $[r, +\infty)$ استطيع عبرت زير السجل و جعل المبره و $a=0$ است

$$\begin{cases} (r, 0) \rightarrow rb + c = 0 \\ (r, 1) \rightarrow rb + c = 1 \end{cases} \rightarrow b = -1 \rightarrow c = 1$$

$$g(n) = \sqrt{bn^2 + an + c} \rightarrow g(n) = \sqrt{x + f} \rightarrow \begin{cases} \frac{-b}{2a} = 0 \\ \text{Ext} = r \end{cases} \rightarrow R_g: \sqrt{[r, +\infty)} = [r, +\infty)$$

$y_{\min} = f$

2
7

$$f(n) = \frac{x^2 - x_n}{x + r} = \frac{x(n-r)(n+r)}{x+r} = x(n-r) = x^2 - r_n \xrightarrow{\text{عدا و مبره مع دجينا}} x^2 - r_n \neq 1 \Rightarrow x^2 - r_n - 1 \neq 0$$

$$(n+r)(n-r) \neq 0 \rightarrow \begin{cases} n \neq -r \\ n \neq r \end{cases}$$

$$D_f: \mathbb{R} - \underbrace{\{-r, r\}}_{a, b}$$

$$f(n) = x^2 - r_n \rightarrow y_{\min} = \frac{-a}{4a} = \frac{-(1-r(-1))}{4} = -1 \Rightarrow R_f: [-1, +\infty) - \{1\}$$

$$f\left(\frac{a+b}{rc}\right) = f\left(\frac{-r+1}{-r}\right), f(-1) = (-1)^2 - r(-1) = 1 + r = 3 \quad \text{✓}$$

2
8

$$y = \sqrt{x^2 + ax^2 + bx - 1} = \sqrt{(n-r)^2} = \sqrt{x^2 - 4x + 12m - 1} \rightarrow a = -4, b = 12$$

$$\Rightarrow D_f = [-4, 12]$$

$$x = -4 \rightarrow y = \sqrt{(-4-r)^2} = -1$$

$$x = 12 \rightarrow y = \sqrt{(12-r)^2} = 10 \rightarrow R_f = [-1, 10] \quad \text{✓}$$

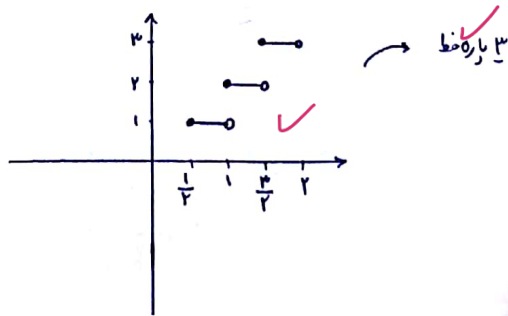
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9

$$y: [n] \rightarrow \left[x + \frac{1}{r}\right]$$

$$\frac{1}{r} \leq x < 1 \rightarrow y = 1$$

$$1 \leq x < \frac{r}{r} \rightarrow y = 2$$

$$\frac{r}{r} \leq x < 2 \rightarrow y = 3$$



2
10