

الف) $f(x) = 2\sin^2(x + \frac{\pi}{4}) - 1 \rightarrow 0 \leq \sin^2(x + \frac{\pi}{4}) \leq 1 \rightarrow 0 \leq 2\sin^2(x + \frac{\pi}{4}) \leq 2 \rightarrow -1 \leq 2\sin^2(x + \frac{\pi}{4}) - 1 \leq 1$

$\Rightarrow R_f = [-1, 1]$

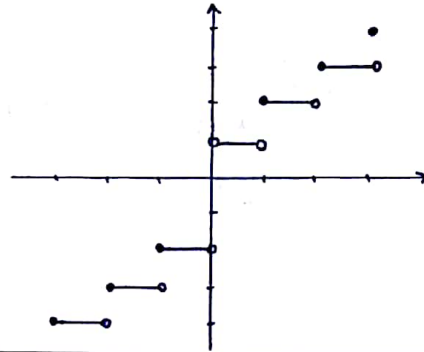
ب) $g(x) = 4\cos^2 x - 4 \rightarrow -1 \leq \cos^2 x \leq 1 \rightarrow -4 \leq 4\cos^2 x \leq 4 \rightarrow -4 \leq 4\cos^2 x - 4 \leq 0$

$\Rightarrow R_g = [-4, 0]$

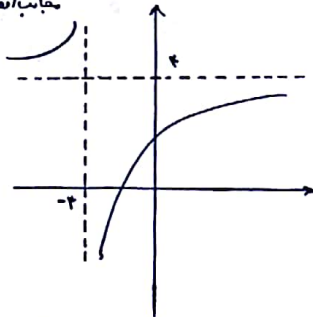
$y = \frac{x}{|x|} + [x]$

$y = \begin{cases} [x] + 1 & x > 0 \\ [x] - 1 & x < 0 \end{cases}$

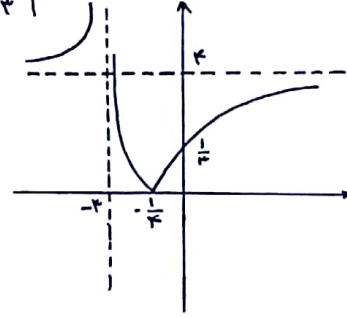
$0 < x < 1 \rightarrow 1$
 $1 \leq x < 2 \rightarrow 2$
 $2 \leq x < 3 \rightarrow 3$
 $x \geq 3 \rightarrow 4$
 $-1 \leq x < 0 \rightarrow -1$
 $-2 \leq x < -1 \rightarrow -2$
 $-3 \leq x < -2 \rightarrow -3$



$y = \frac{x+1}{x+3} \rightarrow \begin{cases} -3: \text{میانگین} \\ 3: \text{میانگین} \end{cases}$



$y = \left| \frac{x+1}{x+3} \right|$



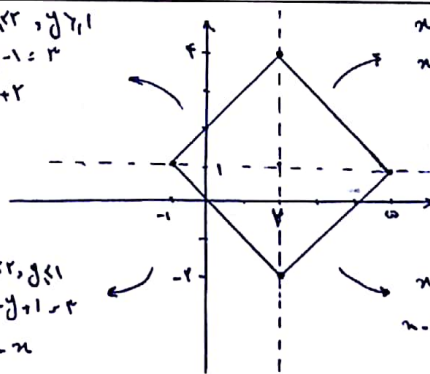
$|x-2| + |y-1| = 3$

$x \geq 2, y \geq 1$
 $-x+2+y-1=3 \rightarrow y=x+2$

$\rightarrow y = x+2$

$x \geq 2, y \leq 1$
 $-x+2-y-1=3 \rightarrow y=-x$

$\rightarrow y = -x$



$x \leq 2, y \geq 1$
 $x-2+y-1=3 \rightarrow y=-x+6$

$\rightarrow y = -x+6$

$x \leq 2, y \leq 1$
 $x-2-y-1=3 \rightarrow y=x-6$

$\rightarrow y = x-6$

الف) $f(x) = x + \frac{1}{x} + 2\sqrt{x + \frac{1}{x}}$

$\sqrt{x + \frac{1}{x}} \geq 0 \rightarrow \begin{cases} \min \square, \sqrt{2} \rightarrow f(x) = 2 + 2\sqrt{2} \\ \max \square \leq +\infty \rightarrow f(x) \leq +\infty \\ -\frac{b}{2a} = -1 \text{ GÜE} \end{cases}$

$\rightarrow R_f = [2 + 2\sqrt{2}, +\infty)$

ب) $y = (2 - \sin x)(1 + \sin x) = 2 + 2\sin x - \sin x - \sin^2 x = -\sin^2 x + 2\sin x + 2$

$\sin x \in \square \rightarrow \begin{cases} \min \square = -1 \rightarrow y = 0 \\ \max \square = 1 \rightarrow y = 4 \\ -\frac{b}{2a} = 1 \rightarrow y = 4 \end{cases} \rightarrow R_g = [0, 4]$

$$f(n) = -\frac{1}{n}n + 10 \rightarrow D_f = \{1, 2, \dots, 9\}$$

$$f(1) + f(2) + f(3) + \dots + f(9) = \left(-\frac{1}{1}(1) + 10\right) + \left(-\frac{1}{2}(2) + 10\right) + \left(-\frac{1}{3}(3) + 10\right) + \dots + \left(-\frac{1}{9}(9) + 10\right) =$$

$$-\frac{1}{1}(1+2+3+\dots+9) + \underbrace{(9 \times 10)}_{\frac{9 \times 10}{2} = 45} = -10 + 45 = \boxed{35}$$

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$$f(x) = \sqrt{ax^2 + bx + c} \quad D_f = [2, +\infty)$$

حیث دامنه صورت $[2, +\infty)$ است یعنی عبارت زیر رادیکال در آن مرتبه و $a=0$ است

$$\begin{cases} (2, 0) \rightarrow 2b + c = 0 \\ (3, 1) \rightarrow 3b + c = 1 \end{cases} \rightarrow b = 1 \rightarrow c = -2$$

$$g(x) = \sqrt{bx^2 + ax + c} \rightarrow g(x) = \sqrt{x^2 + 1} \rightarrow \begin{cases} \frac{-b}{2a} = 0 \\ \text{در } x=0 \end{cases} \rightarrow R_g = \sqrt{[2, +\infty)} = \boxed{[2, +\infty)}$$

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$$f(x) = \frac{x^2 - x}{x+2} = \frac{x(x-2)(x+2)}{x+2} = x(x-2) = x^2 - 2x \xrightarrow{\text{در } x=1 \text{ و } x=2 \text{ به صفر می رسد}} x^2 - 2x + 1 = 0 \Rightarrow x^2 - 2x - 1 = 0$$

$$(x+2)(x-4) \neq 0 \rightarrow \begin{cases} x \neq -2 \\ x \neq 4 \end{cases}$$

$$D_f = \mathbb{R} - \underbrace{\{-2, 4\}}_{a, b}$$

$$f(x) = x^2 - 2x \rightarrow y_{\min} = \frac{-\Delta}{4a} = \frac{-(4-4)}{4} = -1 \Rightarrow R_f = \boxed{[-1, +\infty) - \{1\}}$$

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$$y = \sqrt{x^2 + ax^2 + bx - 1} = \sqrt{(x-2)^2} = \sqrt{x^2 - 4x + 4 - 1} \rightarrow a = -4, b = 11$$

$$\Rightarrow D_f = [-4, 11]$$

$$x = -4 \rightarrow y = \sqrt{(-4-2)^2} = -6$$

$$x = 11 \rightarrow y = \sqrt{(11-2)^2} = 9$$

$$\rightarrow R_f = \boxed{[-6, 9]}$$

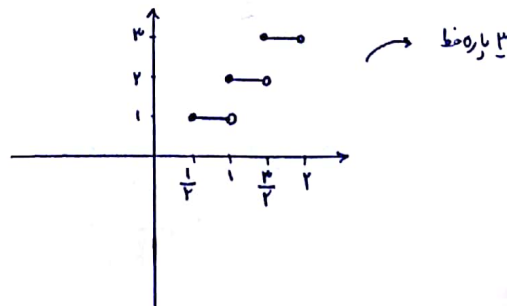
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$$y = [n] + \left[x + \frac{1}{x}\right]$$

$$\frac{1}{x} < 1 \rightarrow y = 1$$

$$1 < x < \frac{x}{2} \rightarrow y = 2$$

$$\frac{x}{2} < x < 2 \rightarrow y = 3$$



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