

1) $-1 \leq \sin t \leq 1 \rightarrow -1 \leq \sin\left(\frac{\pi}{3} + 2\pi\right) \leq 1 \rightarrow 0 \leq \sin^2\left(2\pi + \frac{\pi}{3}\right) \leq 1 \rightarrow$

$0 \leq r \sin^2\left(2\pi + \frac{\pi}{3}\right) \leq r \rightarrow -1 \leq r \sin^2\left(2\pi + \frac{\pi}{3}\right) \leq 1 \quad R = [-1, 1]$

2) $-1 \leq \cos \theta \leq 1 \rightarrow (-1)^3 \leq \cos^3(\theta) \leq 1^3 \rightarrow -1 \leq \cos^3 \theta \leq 1 \rightarrow -3 \leq r \cos^3 \theta \leq 3 \rightarrow$

$-3 \leq r \cos^3 \theta \leq 3 \rightarrow R = [-3, 3]$

$-3 \leq x < -2 \Rightarrow -1 - 3 = -4$

$-2 \leq x < -1 \Rightarrow -1 - 2 = -3$

$-1 \leq x < 0 \Rightarrow -1 - 1 = -2$

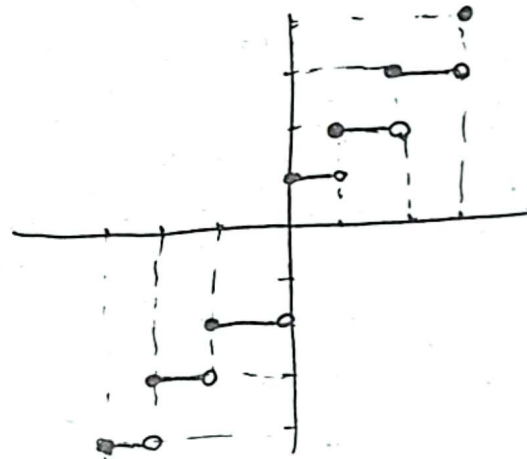
$x = 0$

$0 \leq x < 1 \Rightarrow 1 + 0 = 1$

$1 \leq x < 2 \Rightarrow 1 + 1 = 2$

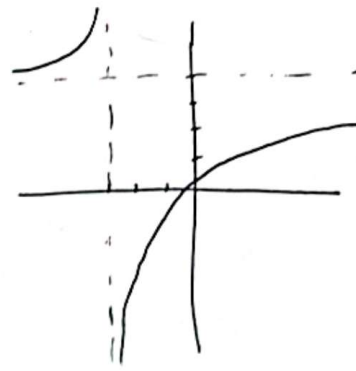
$2 \leq x < 3 \Rightarrow 2 + 1 = 3$

$x = 3 \Rightarrow 1 + 3 = 4$

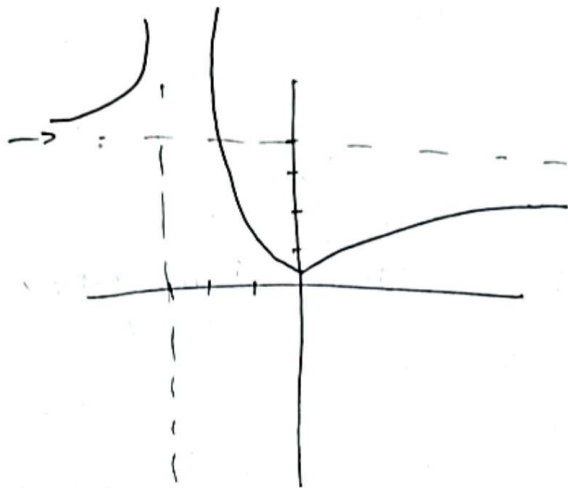


$$y = \left| \frac{x+1}{2+r} \right|$$

$$\frac{x+1}{2+r} \rightarrow$$



نقطه ۱



$$|x-1| + |y-1| = r$$

$$y \geq 1, x < 1$$

$$-x+r+y-1=r$$

$$y=x+1$$

$$y \geq 1, x \geq 1$$

$$x-r+y-1=r \rightarrow y=-x+r$$

$$-x+r-y+1=r$$

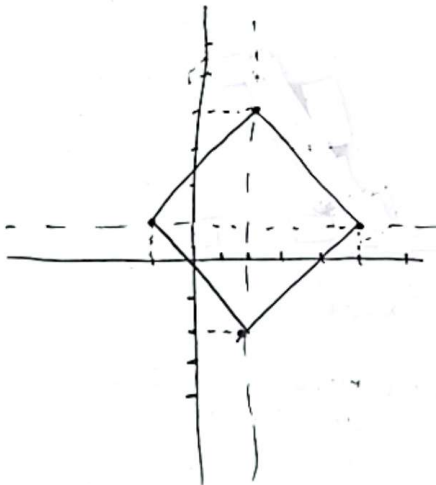
$$y=-x$$

$$y < 1, x < 1$$

$$x-r-y+1=r$$

$$y=x-1$$

$$y < 1, x \geq 1$$



$$\omega) 2 + \frac{1}{x} + x\sqrt{2 + \frac{1}{x}}$$

$$2 + \frac{1}{x} \geq 0 \rightarrow \underbrace{2 + \frac{1}{x}}_{D=(0, +\infty)} \geq x \xrightarrow{x(-)} -\left(2x + \frac{1}{x}\right) \leq -x$$

$$2 + \frac{1}{x} = t \rightarrow t + x\sqrt{t} : \text{بحسب } (t \geq 2)$$

$$t + x\sqrt{t} \rightarrow \text{الحد الأدنى } \frac{y_{\min}}{t \min = 2} \rightarrow 2 + x\sqrt{2} = y_{\min} \quad \lim_{t \rightarrow +\infty} (t + x\sqrt{t}) = +\infty$$

$$R = [2 + x\sqrt{2}, +\infty)$$

$$\text{ـ) } y = (x - \sin x)(1 + \sin x) = -\sin^2 x - \sin x + x \sin x + x = -\sin^2 x + x \sin x + x$$

$$\begin{aligned} \min \quad \min x = -1 &\rightarrow -1 - x + x = 0 \\ \xrightarrow{-\frac{b}{2a}} &-\frac{x}{-1} = 1 \rightarrow -1 + x + x = x \\ \max \quad 1 &\rightarrow x \end{aligned}$$

$$R = [0, x]$$

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$$f(x) = -\frac{1}{x} x + 1$$

$$\begin{aligned} 1 &\rightarrow \frac{x}{x} \\ x &\rightarrow \frac{x}{x} \\ x &\rightarrow \frac{x}{x} \\ x &\rightarrow \frac{x}{x} \\ x &\rightarrow \frac{x}{x} \\ x &\rightarrow \frac{x}{x} \end{aligned}$$

$$\begin{aligned} v &\rightarrow \frac{x}{x} \\ \wedge &\rightarrow \frac{x}{x} \\ q &\rightarrow \frac{x}{x} \end{aligned}$$

$$\frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9}{x} = \frac{x x_5}{x} = x_5$$

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$$a > 0$$

$$x = x \rightarrow \text{نقطة الحد الأدنى}$$

$$\frac{x}{-1 +}$$

$$ax^2 + bx + c = a(x - x)^2 = a(x^2 - \epsilon x + \epsilon) = ax^2 - \epsilon ax + \epsilon a$$

$$b = -\epsilon a, c = \epsilon a \xrightarrow{f(x)=1} \sqrt{9a + x^2 b + c} = 1 \rightarrow 9a + x^2 b + c = 1 \rightarrow a = 1, b = -\epsilon, c = \epsilon$$

$$\sqrt{bx^2 + ax - c} = \sqrt{-\epsilon x^2 + x - 1}$$

$$-\epsilon x^2 + x - 1 \geq 0 \Rightarrow \epsilon x^2 - x + 1 \leq 0$$

$$\Delta = b^2 - 4ac = 1 - 4(\epsilon)(1) = -4\epsilon$$

$$\Delta < 0, a < 0$$

الحد الأدنى ← تقريباً

$$\frac{2^x - \varepsilon x}{x + 2} \quad D = \mathbb{R} - \{a, b\} \quad f\left(\frac{a+b}{2c}\right) = ?$$

$$R = [c, +\infty) - \{1\}$$

$$\frac{2^x - \varepsilon x}{x + 2} \rightarrow \frac{2(2^x - 2)}{2 + 2} = \frac{2(2^x - 2)(2 + 2)}{2 + 2}, \quad 2^x \neq 2 \text{ بشي} \rightarrow 2^x - 2x$$

$$a = b = -2$$

$$f(x) = 2^x - 2x$$

$$0^1 \rightarrow x = -\frac{b}{a} = \frac{2}{1} = 2 \quad y = -1$$

$$f(-2) = \varepsilon + \varepsilon = 1 \quad \text{ریشه منفي}$$

$$R = [-1, +\infty) - \{1\} \rightarrow c = -1$$

$$f\left(\frac{a+b}{2c}\right) = f\left(\frac{-2}{-2}\right) = f(1) = 2^1 - 2(1) = 0$$

$$y = \sqrt[3]{2^x + a2^x + b2^x - 1} \rightarrow \sqrt[3]{(2-x)^3} = 2-x$$

$$(2-x)^3 \rightarrow (2-x)^3 = 2^3 - 4x2^2 + 12x^2 - 1 \rightarrow b = 12, a = -4$$

$$D = [-4, 12] \quad \left. \begin{array}{l} \rightarrow x = -4 \rightarrow -4 - x = -1 \\ \rightarrow x = 12 \rightarrow 12 - x = 1 \end{array} \right\} R = [-1, 1]$$

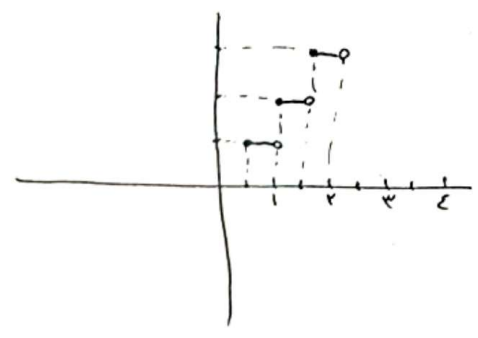
$$y = [x] + [x + \frac{1}{2}] \quad [\frac{1}{2}, 2)$$

$$0 \leq x < \frac{1}{2} \rightarrow [x] = 0, \quad \frac{1}{2} \leq x + \frac{1}{2} < 1 \rightarrow [x + \frac{1}{2}] = 0 \rightarrow f(x) = 0 + 0 = 0$$

$$\frac{1}{2} \leq x < 1 \rightarrow [x] = 0, \quad 1 \leq x + \frac{1}{2} < \frac{3}{2} \rightarrow [x + \frac{1}{2}] = 1 \rightarrow f(x) = 0 + 1 = 1$$

$$1 \leq x < \frac{3}{2} \rightarrow [x] = 1, \quad \frac{3}{2} \leq x + \frac{1}{2} < 2 \rightarrow [x + \frac{1}{2}] = 1 \rightarrow f(x) = 1 + 1 = 2$$

$$\frac{3}{2} \leq x < 2 \rightarrow [x] = 1, \quad 2 \leq x + \frac{1}{2} < \frac{5}{2} \rightarrow [x + \frac{1}{2}] = 2 \rightarrow f(x) = 1 + 2 = 3$$



به 0, 1, 2, 3