۱- وبیناوونی

الف) $\lim\limits_{x\to 2^+} \dfrac{3x+5}{x+1} = \dfrac{3\times 2+5}{2+1} = 3$

ب) $\lim\limits_{x\to 2^-} \dfrac{3x+5}{x+1} = 3$

ج) $\lim \dfrac{3x+5}{x+1} = \dfrac{3\times 2+5}{2+1} = 3$

د) $\lim\limits_{x\to 0} \dfrac{3x+7}{[x^2]} =$ تعریف نشده

$0^+ \to [0^+] = 0$

۲-

الف) $\lim\limits_{x\to 3^-} 4[x]-2 = 4\times 2-2 = 6$

$x<3 \to [x]=2$

ب) $\lim\limits_{x\to 3^-} [4x-2] = 9$

$x<3 \to 4x-2<10$

پ) $\left[\lim\limits_{x\to 3^-} 4x-2\right] = 10$

$\lim\limits_{x\to 3^-} 4x-2 = 10$

د) $\left[\lim\limits_{x\to 3^+} 4x-2\right] = 10$

$\lim\limits_{x\to 3^+} 4x-2 = 10$

۳-

الف) $\lim\limits_{x\to 3^-} \dfrac{3[x]+1}{2} = \dfrac{7}{2} = 3/5$

$x<3 \to [x]=2$

ب) $\lim\limits_{x\to 3^-} \left[\dfrac{3x+1}{2}\right] = 4$

$x<3 \to 3x+1<10 \to \dfrac{3x+1}{2} < 5$

ج) $\left[\lim\limits_{x\to 3^-} \dfrac{3x+1}{2}\right] = 5$

$\lim\limits_{x\to 3^-} \dfrac{3x+1}{2} = 5$

د) $\left[\lim\limits_{x\to 3^+} \dfrac{3x+1}{2}\right] = 5$

$\lim\limits_{x\to 3^+} \dfrac{3x+1}{2} = 5$

-۴

الف) $\lim_{x\to 1} \frac{x-5}{(x-1)(x-5)} = \lim_{x\to 1} \frac{1}{x-1} \longrightarrow \lim_{x\to 1^+} \frac{1}{x-1} = \lim \frac{1}{0^+} = +\infty$

$\searrow \lim_{x\to 1^-} \frac{1}{x-1} = \lim \frac{1}{0^-} = -\infty$

ب) $\lim_{x\to 3} \frac{x-4}{(x-3)^2} = \frac{-1}{\text{همواره } 0^+} = -\infty$

-۵

الف) $x<-3 \Rightarrow -3x>9 \quad 5x<-15 \quad [-3x]=9 \quad [5x]=-16 \Big\} \Rightarrow \lim_{x\to -3^-} f(x) = 25$

$x>-3 \quad -3x<9 \quad 5x>-15 \quad [-3x]=8 \quad [5x]=-15 \Big\} \Rightarrow \lim_{x\to -3^+} f(x) = 8+15=23$

ب) $\lim_{x\to (-\frac{1}{2})^-} \left[\frac{-1}{x^4}\right] = -16$

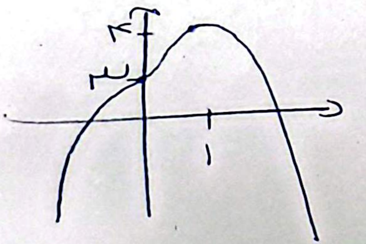
$x<-\frac{1}{2} \Rightarrow x^2>\frac{1}{4} \Rightarrow x^4>\frac{1}{16} \quad \frac{1}{x^4}<16$

$\Rightarrow -\frac{1}{x^4}>-16$

-۶

الف) $f'(x)=12x^2-12x^3=12x^2(1-x)$

$+ \quad 0 \quad + \quad 1 \quad -$



$f(1)=4$
$f(0)=3$

$\lim_{x\to 1} [f(x)] = 3$

ب)

از شکل مورد الف استفاده می‌کنیم طبق شکل

$\lim_{x\to 0^+} [f(x)] = 3 \qquad \lim_{x\to 0^-} [f(x)] = 2$

۷-

الف) $\lim_{x\to 2^+} \frac{|x^3-8|}{x^2-4} = \frac{x^3-8}{x^2-4} = \frac{(x-2)(x^2+2x+4)}{(x-2)(x+2)} = \frac{x^2+2x+4}{x+2} = \frac{12}{4} = 3$

$x\to 2^- \Rightarrow \frac{-(x^3-8)}{x^2-4} = -3$

ب) $\lim_{x\to\pi} \frac{|\sin x|}{\sin 2x}$

$x\to\pi^+ \quad \frac{-\sin x}{2\sin x\cos x} = -\frac{1}{2\cos x} = \frac{1}{2}$

$x\to\pi^- \quad \frac{\sin x}{2\sin x\cos x} = \frac{1}{2\cos x} = -\frac{1}{2}$

۸-

الف) $\lim_{x\to \frac{3\pi}{2}} \sqrt{\cos x}$

$x\to \frac{3\pi}{2}^+ \Rightarrow \cos x>0 \quad \lim\sqrt{0}=0$

$x\to \frac{3\pi}{2}^- \Rightarrow \cos x<0 \Rightarrow$ تعریف نشده

ب) $\lim_{x\to 0} \sqrt{x-2\sqrt{x}}$

$x\to 0^+ \Rightarrow 0$

$x\to 0^- \Rightarrow$ تعریف نشده

۹-

الف) $\lim_{x\to 0^+} \frac{(-1)^{[\sin x]}}{x^3-x^2}$

$x\to 0^+ \Rightarrow [\sin x]=0$

$= \frac{1}{x^3-x^2} = \frac{1}{x^2(x-1)} = -\infty$

به ازای 0^+ عبارت منفی است

$x\to 0^- \Rightarrow [\sin x] = -1 \Rightarrow \frac{-1}{x^3-x^2} = \frac{-1}{x^2(x-1)} = +\infty$

به ازای 0^- عبارت منفی است

ب) $\lim \frac{x^2-[x^2]}{x-[x]}$

$x\to 2^+ \Rightarrow x>2 \Rightarrow x^2>4 \Rightarrow [x^2]=4$

$[x]=2$

$\lim_{x\to 2^+} \frac{x^2-4}{x-2} = x+2 = 4$

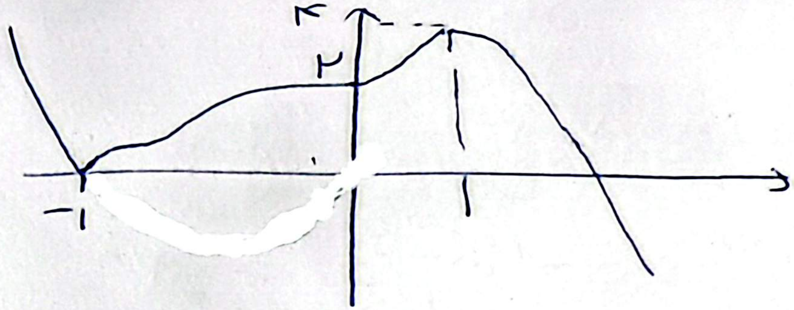
$x\to 2^- \quad x<2 \Rightarrow x^2<4 \Rightarrow [x^2]=3$

$[x]=1$

$\lim \frac{x^2-3}{x-1} = \frac{1}{1}$

الف) $f'(x) = 15x^2 - 15x^4 = \quad 15x^2(1-x^2)$

$-$	-1	$+$	0	$+$	1	$-$

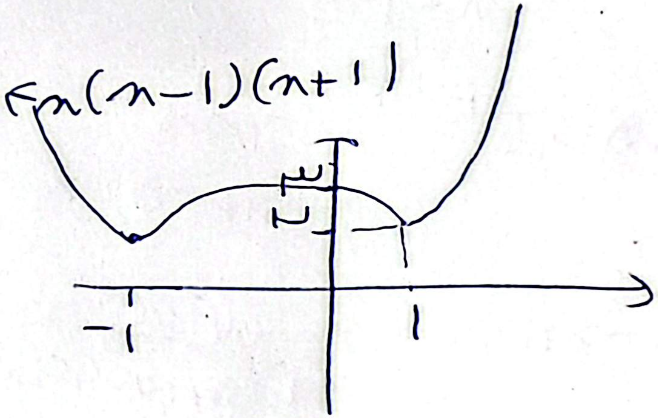


$f(0)=2 \quad f(1)=5-3+2=4$

$f(-1)=0$

ب) $f'(x) = 4x^3 - 4x = 4x(x-1)(x+1)$

$-$	-1	$+$	0	$-$	1	$+$



$f(0)=3 \quad f(1)=2$

$f(-1)=1-2+3=2$