مهرنیا ریاضی   دوازدهم دختر B

1

$$\lim_{x\to 2}\frac{2x-5}{x^2+ax+b}=-\infty \longrightarrow (x-2)^2=x^2-4x+4 \qquad \begin{matrix} a=-4\\ b=4\end{matrix} \qquad a+b=0$$

2

$$\lim_{x\to\sqrt[3]{4}}\frac{x}{x^2+ax+b}=+\infty \longrightarrow (x-\sqrt[3]{4})^2=x^2-2\sqrt[3]{4}x+\sqrt[3]{16}=x^2-2\sqrt[3]{4}x+\sqrt[3]{16}$$

$$a=-2\sqrt[3]{4} \qquad b=\sqrt[3]{16} \qquad \left[\frac{b}{a}\right]=\left[\frac{\sqrt[3]{16}}{-2\sqrt[3]{4}}\right]=-1$$

3

$$\lim \frac{[-x]+a}{\left|\frac{\sqrt{3}}{3}x+a\right|+a}=-\infty \qquad \frac{-1+a}{\left|\frac{1}{3}+a\right|+a} \qquad a-1<0 \quad a<1$$

$$\longrightarrow \frac{1}{3}+2a=0 \qquad a=\frac{-1}{6} \qquad [a]=-1$$

4

$$\lim_{x\to(-\frac{1}{2})^+}\frac{14x-\left[-\frac{2}{x^2}\right]}{24x+\left[\frac{3}{x^2}\right]} \qquad x>-\frac{1}{2} \qquad x^2<\frac{1}{4} \qquad \frac{1}{x^2}>4 \qquad \frac{-2}{x^2}<-8$$

$$\frac{3}{x^2}>12$$

$$\lim_{x\to(-\frac{1}{2})^+}\frac{14x+9}{24x+12}=\frac{-8+9}{12(2x+1)}=\frac{1}{12\times 0^+}=+\infty$$

5

$$\lim_{x\to 2^+}\frac{x^2-4}{x^3-[x^3]} \qquad x>2 \qquad x^3>8$$

$$\lim\frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(x^2+2x+4)}=\frac{(x+2)}{x^2+2x+4}=\frac{4}{12}=\frac{1}{3}$$

6

$$\lim_{x\to -8}\frac{x^2+10x+16}{12+6\sqrt[3]{x}}=\frac{(x+2)(x+8)}{6(2+\sqrt[3]{x})}\times\frac{(4-2\sqrt[3]{x}+\sqrt[3]{x^2})}{(4-2\sqrt[3]{x}+\sqrt[3]{x^2})}=\frac{(x+2)(4-2\sqrt[3]{x}+\sqrt[3]{x^2})}{6}=$$

$$\frac{(-6)(12)}{6}=-12$$

7

$$\lim_{x\to 0^-}\frac{\sqrt{2+3x}-\sqrt{2-x}}{\sqrt{1-\cos x}}\times\frac{\sqrt{2+3x}+\sqrt{2-x}}{\sqrt{2+3x}+\sqrt{2-x}}=\frac{4x}{\frac{|x|}{\sqrt{2}}\left(\sqrt{2+3x}+\sqrt{2-x}\right)}=\frac{-4\sqrt{2}}{2\sqrt{2}}=-2$$

$$1-\cos x \sim 1-\frac{x^2}{2} \qquad \sqrt{1-\cos x}=\sqrt{1-(1-\frac{x^2}{2})}=\frac{|x|}{\sqrt{2}}$$

8

$$\lim_{x\to 0}\frac{k+\cos(\sqrt{a}x)}{kx^2}=3 \longrightarrow \frac{k+(1-\frac{ax^2}{2})}{kx^2}=\frac{-1+1-\frac{ax^2}{2}}{-x^2}=\frac{a}{2}=3$$

$$k+\cos(0)=0 \qquad k+1=0 \qquad k=-1$$

$$\boxed{a=6}$$

$$\lim_{x \to 3a^+} \frac{2\sqrt{x-3a} + 3\sqrt{x} - \sqrt{27a}}{\sqrt{x^2 - 9a^2}} = \frac{2\sqrt{x-3a} + 3(\sqrt{x} - \sqrt{3a})}{(\sqrt{x-3a})(\sqrt{x+3a})} \times \frac{\sqrt{x}+\sqrt{3a}}{\sqrt{x}+\sqrt{3a}}$$ 9

$$\frac{(2\sqrt{x-3a})(\sqrt{x}+\sqrt{3a}) + 3(x-3a)}{(\sqrt{x-3a})(\sqrt{x+3a})(\sqrt{x}+\sqrt{3a})} = \frac{\cancel{(\sqrt{x-3a})}(2\sqrt{x} + 2\sqrt{3a} + 3\sqrt{x-3a})}{\cancel{(\sqrt{x-3a})}(\sqrt{x+3a})(\sqrt{x}+\sqrt{3a})}$$

$$\frac{2\sqrt{3a}+2\sqrt{3a}+0}{2\sqrt{3a}\times\sqrt{6a}} = \frac{4\sqrt{3a}}{2\sqrt{3a}\times\sqrt{6a}} = \frac{2}{\sqrt{6a}}$$

10

$$\lim_{x \to -1} \frac{1 - k[x]}{x^2 - 1} = -\infty$$

-1^+: $\quad \dfrac{1+k}{(x-1)(x+1)} = -\infty \qquad 1+k>0 \qquad k>-1$

-1^-: $\quad \dfrac{1+2k}{(x-1)(x+1)} = -\infty \qquad 1+2k<0 \qquad k<\dfrac{-1}{2}$

$$-1 < k < \frac{-1}{2} \longrightarrow -\pi < k\pi < \frac{-\pi}{2}$$

$$\cos(-\pi) < \cos k\pi < \cos\left(\frac{-\pi}{2}\right) \longrightarrow \boxed{-1 < \cos k\pi < 0}$$

تغییر سو