مبینا جعفری پور — تکلیف ۲۰ — دوازدهم دختر ۵

۱ $\lim\limits_{x\to 2}\dfrac{2x-5}{x^2+ax+b}=-\infty \Rightarrow (x-2)^2=x^2+ax+b=x^2-4x+4 \Rightarrow \begin{cases} a=-4 \\ b=4 \end{cases} \Rightarrow \boxed{a+b=0}$

از آنجایی که صورت در همسایگی $x=2$ هم علامت است پس مخرج ریشه مضاعف داشته است.

۲ $\lim\limits_{x\to(\sqrt[3]{4})}\dfrac{x}{x^2+ax+b}=+\infty \Rightarrow (x-\sqrt[3]{4})^2=x^2+ax+b=x^2-2\sqrt[3]{4}\,x+\sqrt[3]{16} \Rightarrow \begin{cases} a=-2\sqrt[3]{4} \\ b=\sqrt[3]{16} \end{cases} \Rightarrow \left[\dfrac{b}{a}\right]=\left[\dfrac{\sqrt[3]{16}}{-2\sqrt[3]{4}}\right]=\boxed{-1}$

۳ $\lim\limits_{x\to(\frac{1}{\sqrt{3}})^+}\dfrac{[-x]+a}{\left|\frac{\sqrt{3}}{3}x+a\right|+a}=\lim\limits_{x\to(\frac{1}{\sqrt{3}})^+}\dfrac{a-1}{\left|\frac{1}{3}+a\right|+a}=-\infty \Rightarrow \begin{cases} \frac{1}{3}+a+a=0 \Rightarrow a=-\frac{1}{6} \Rightarrow [a]=\boxed{-1} \\ -(\frac{1}{3}+a)+a=0 \Rightarrow -\frac{1}{3} \text{ غ ق ق} \end{cases}$

۴ $\lim\limits_{x\to(-\frac{1}{2})^+}\dfrac{16x-\left[-\frac{2}{x^2}\right]}{24x+\left[\frac{3}{x^2}\right]}=\lim\limits_{x\to(-\frac{1}{2})^+}\dfrac{-8-(-9)}{12+(12)}=\boxed{\dfrac{1}{24}}$

۵ $\lim\limits_{x\to 2^+}\dfrac{x^2-4}{x^3-[x^3]}=\lim\limits_{x\to 2^+}\dfrac{x^2-4}{x^3-8}\overset{\frac{0}{0}}{=}\lim\limits_{x\to 2^+}\dfrac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(x^2+2x+4)}=\boxed{\dfrac{1}{3}}$

۶ $\lim\limits_{x\to -8}\dfrac{x^2+10x+16}{12+6\sqrt[3]{x}}\overset{\frac{0}{0}}{=}\lim\limits_{x\to -8}\dfrac{\cancel{(\sqrt[3]{x}+2)}(\sqrt[3]{x^2}-2\sqrt[3]{x}+4)(x+2)}{6\cancel{(2+\sqrt[3]{x})}}=\boxed{-12}$

۷ $\lim\limits_{x\to 0^-}\dfrac{\sqrt{2+3x}-\sqrt{2-x}}{\sqrt{1-\cos x}}\overset{\frac{0}{0}}{=}\lim\limits_{x\to 0^-}\dfrac{\sqrt{2+3x}-\sqrt{2-x}}{\underbrace{\sqrt{1-(1-\frac{x^2}{2})}}_{\text{هم ارزی}}}\times\dfrac{\sqrt{2+3x}+\sqrt{2-x}}{\sqrt{2+3x}+\sqrt{2-x}}=\lim\limits_{x\to 0^-}\dfrac{4x}{\frac{|x|}{\sqrt{2}}\times 2\sqrt{2}}=\lim\limits_{x\to 0^-}\dfrac{4x}{-x\times 2}=\boxed{-2}$

۸ $\lim\limits_{x\to 0}\dfrac{K+\cos(\sqrt{a}x)}{Kx^2}=3 \xrightarrow[\text{صورت هم صفر بوده}]{\text{مخرج صفر شده پس}} K+\cos(\sqrt{a}(0))=0=K+1 \Rightarrow \boxed{K=-1}$

$\lim\limits_{x\to 0}\dfrac{-1+\cos(\sqrt{a}x)}{-x^2}\overset{\text{هم ارزی}}{=}\lim\limits_{x\to 0}\dfrac{-1+(1-\frac{ax^2}{2})}{-x^2}=\lim\limits_{x\to 0}\dfrac{-\frac{ax^2}{2}}{-x^2}=\dfrac{a}{2}=3 \Rightarrow \boxed{a=6}$

$\Rightarrow \dfrac{a}{K}=\dfrac{6}{-1}=\boxed{-6}$

۹ $\lim\limits_{x\to(3a)^+}\dfrac{2\sqrt{x-3a}+3\sqrt{x}-\sqrt{27a}}{\sqrt{x^2-9a^2}}\overset{\frac{0}{0}}{=}\lim\limits_{x\to(3a)^+}\dfrac{2\sqrt{x-3a}}{\sqrt{x-3a}\times\sqrt{x+3a}}+\dfrac{3(\sqrt{x}-\sqrt{3a})}{\sqrt{x-3a}\times\sqrt{x+3a}}\times\dfrac{\sqrt{x}+\sqrt{3a}}{\sqrt{x}+\sqrt{3a}}=\lim\limits_{x\to(3a)^+}\dfrac{2}{\sqrt{6a}}+\dfrac{3\overset{\sqrt{x-3a}}{\cancel{(x-3a)}}}{\cancel{\sqrt{x-3a}}\times\sqrt{x+3a}\times 2\sqrt{3a}}=\dfrac{2}{\sqrt{6a}}+0=\boxed{\dfrac{2}{\sqrt{6a}}}$

۱۰ $\lim\limits_{x\to(-1)}\dfrac{1-K[x]}{x^2-1}=\begin{cases} \lim\limits_{x\to(-1)^+}\dfrac{1-K(-1)}{0^-}=-\infty \Rightarrow 1+K>0 \Rightarrow K>-1 \\ \lim\limits_{x\to(-1)^-}\dfrac{1-K(-2)}{0^+}=-\infty \Rightarrow 1+2K<0 \Rightarrow K<\frac{-1}{2} \end{cases} \Rightarrow -1<K<-\frac{1}{2} \Rightarrow \begin{cases} -\pi<K\pi<-\frac{\pi}{2} \\ -1<\cos K\pi<0 \end{cases} \Rightarrow x<0,\ y<0$ $\boxed{\text{ناحیه سوم}}$