۲۰ آفرین

باران تسلیمی - تکلیف ۲۰ -

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$$\lim_{x\to 2} \frac{2x-5}{x^2+ax+b} = -\infty \xrightarrow{x\to 2} \frac{-1}{\underbrace{4+2a+b}_{0^+}} = -\infty$$

$(I)\ 2a+b+4=0 \longrightarrow b=-2a-4$

$(II) \longrightarrow \Delta=0 \rightsquigarrow a^2-4b=0 \rightsquigarrow a^2+8a+16=0$

$(a+4)^2=0 \longrightarrow \boxed{a=-4}$

$b=-2(-4)-4=+4$

$\boxed{b=+4}$

$a+b=-4+4=\underline{\underline{0}}$

$$\lim_{x \to \sqrt[3]{4}} \frac{x}{x^2+ax+b} = \lim_{x \to \sqrt[3]{4}} \frac{\sqrt[3]{4}}{x^2+ax+b} \leadsto x^2+ax+b = (x-\sqrt[3]{4})^2$$ ۳-

$$\Rightarrow x^2 - 2\sqrt[3]{4}\,x + \sqrt[3]{16} \qquad \begin{cases} a = -2\sqrt[3]{4} \\ b = \sqrt[3]{16} \end{cases}$$

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$$\left[\frac{b}{a}\right] \Rightarrow \left[-\sqrt[3]{\frac{16}{25}}\right] = -1$$

-۳۰

$$\lim_{x \to \left(\frac{1}{\sqrt{۳}}\right)^+} \frac{[-x] + a}{\left|\frac{\sqrt{۳}}{۳}x + a\right| + a} = -\infty \qquad [a] = ?$$

$$[a] = \left[-\frac{1}{۶}\right] = -1$$

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$$\lim \frac{a-1}{\left|\frac{1}{۳} + a\right| + a} = -\infty$$

$$\left|\frac{1}{۳} + a\right| + a = 0 \rightarrow \frac{1}{۳} + a = -a \rightarrow -۲a = \frac{1}{۳} \rightarrow a = \frac{-1}{۶}$$

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$$\lim_{x\to(-\frac{1}{2})^+} \frac{16x-\left[\frac{-2}{x^2}\right]}{24x+\left[\frac{3}{x^2}\right]}$$

$$x>\frac{-1}{2} \to x^2<\frac{1}{4} \rightsquigarrow \frac{1}{x^2}>4 \rightsquigarrow \frac{-8+9}{-12^+ +12} = \frac{1}{0^+} = +\infty$$

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$$\lim_{x\to 2^+} \frac{x^2-4}{x^3-[x^3]} \xrightarrow{\text{مبهم } \frac{0}{0}} \frac{x^2-4}{x^3-8} = \frac{(\cancel{x-2})(x+2)}{(\cancel{x-2})(x^2+2x+4)} = \frac{4}{12} = \frac{1}{3}$$

۶-

$$\lim_{x\to-8} \frac{x^2+10x+16}{12+6\sqrt[3]{x}} \rightsquigarrow \frac{(x+8)(x+2)}{6(2+\sqrt[3]{x})} \times \frac{(\sqrt[3]{x^2}-2\sqrt[3]{x}+4)}{(\sqrt[3]{x^2}-2\sqrt[3]{x}+4)} = \frac{(\cancel{x+8})(x+2)(\sqrt[3]{x^2}-2\sqrt[3]{x}+4)}{6(\cancel{x+8})}$$

$$\frac{-\cancel{6}(\cancel{4}+\cancel{4}+4)}{\cancel{6}} = \rightsquigarrow -12$$

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$$\lim_{x\to 0^-} \frac{\sqrt{2+3x}-\sqrt{2-x}}{\sqrt{1-\cos x}}$$

$$\times \frac{\sqrt{1+\cos x}}{\sqrt{1+\cos x}} \times \frac{\sqrt{2+3x}+\sqrt{2-x}}{\sqrt{2+3x}+\sqrt{2-x}} \Rightarrow \lim_{x\to 0^-} \frac{(2+3x-2+x)(\sqrt{2})}{(\sqrt{1-\cos^2 x})(2\sqrt{2})} = \frac{4x}{2|\sin x|}$$

$$\rightsquigarrow \frac{2x}{-\sin x} = -2$$

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$\lim k + \cos(\sqrt{a}x) = 0 \longrightarrow k+1 = 0 \rightarrow k = -1$

$$\lim_{x\to 0} \frac{-1+\cos(\sqrt{a}\,x)}{-x^2} \xrightarrow{\text{هم‌ارزی}} \lim_{x\to 0} \frac{-\frac{ax^2}{2}}{-x^2} = \frac{a}{2} \longrightarrow \frac{a}{2} = 2$$

$\boxed{a = 4}$ $\qquad \frac{a}{k} = \frac{4}{-1} = -4$

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$$\lim_{x\to 3a^+} \frac{2\sqrt{x}-3\sqrt{a}}{(\sqrt{x}-\sqrt{3a})(\sqrt{x}+\sqrt{3a})} + \frac{3\sqrt{x}-3\sqrt{3a}}{(\sqrt{x}-\sqrt{3a})(\sqrt{x}+\sqrt{3a})} = \lim_{x\to 3a^+}\left(\frac{2}{\sqrt{4a}} + \frac{3(\sqrt{x}-\sqrt{3a})}{(\sqrt{x}-\sqrt{3a})(\sqrt{x}+\sqrt{3a})} \times \frac{\sqrt{x}+\sqrt{3a}}{\sqrt{x}+\sqrt{3a}}\right)$$ -۹

$$\rightarrow \lim_{x\to 3a^+}\left(\frac{2}{\sqrt{4a}} + \frac{3\sqrt{x-3a}}{(\sqrt{x}+\sqrt{3a})(\sqrt{x}-\sqrt{3a})}\right) \Rightarrow \lim_{x\to 3a^+} \frac{2}{\sqrt{4a}} \Rightarrow \frac{\sqrt{4}}{\sqrt{4a}} = \sqrt{\frac{2}{3a}}$$

ارفاق ۲

$$\lim_{x\to(-1)^-} \frac{1-k[x]}{x^2-1} = \frac{1+2k}{0^+} = -\infty \Rightarrow 1+2k < 0$$ -۱۰

$$\lim_{x\to(-1)^+} \frac{1-k[x]}{x^2-1} \rightarrow \frac{1+k}{0^-} = -\infty \Rightarrow 1+k > 0$$

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$$k < \frac{-1}{2} \quad , \quad k > -1 \Rightarrow -1 < k < \frac{-1}{2}$$

$$\begin{cases} -\pi < k\pi < \frac{-\pi}{2} \\ -1 < \cos k\pi < 0 \end{cases} \Rightarrow$$ طول و عرض منفی

ناحیه سوم