باران تسلیمی - تکلیف ۲۰ -

۱-

$$\lim_{x\to 2}\frac{2x-5}{x^2+ax+b}=-\infty \xrightarrow{x\to 2} \frac{-1}{\underbrace{4+2a+b}_{0^+}}=-\infty$$

$(I)\ 2a+b+4=0 \rightarrow b=-2a-4$

$(II) \rightarrow \Delta=0 \rightsquigarrow a^2-4b=0 \rightsquigarrow a^2+8a+16=0$

$(a+4)^2=0 \rightarrow \boxed{a=-4}$

$b=-2(-4)-4=+4$

$\boxed{b=+4}$

$a+b=-4+4=\underline{\underline{0}}$

۲-

$$\lim_{x\to\sqrt[3]{4}} \frac{x}{x^2+ax+b} = \lim_{x\to\sqrt[3]{4}} \frac{\sqrt[3]{4}}{x^2+ax+b} \rightsquigarrow x^2+ax+b = (x-\sqrt[3]{4})^2$$

$$\Rightarrow x^2 - 2\sqrt[3]{4}\,x + \sqrt[3]{16} \quad \begin{cases} a = -2\sqrt[3]{4} \\ b = \sqrt[3]{16} \end{cases}$$

$$\left[\frac{b}{a}\right] \Rightarrow \left[-\sqrt[3]{\frac{16}{32}}\right] = -1$$

-۴۰

$$\lim_{x \to \left(\frac{1}{\sqrt{۳}}\right)^+} \frac{[-x] + a}{\left|\frac{\sqrt{۳}}{۳} x + a\right| + a} = -\infty \qquad [a] = ?$$

$$[a] = \left[\frac{-1}{۶}\right] = -1$$

$$\lim \frac{a - 1}{\left|\frac{1}{۳} + a\right| + a} = -\infty$$

$$\left|\frac{1}{۳} + a\right| + a = 0 \rightarrow \frac{1}{۳} + a = -a \rightarrow -۲a = \frac{1}{۳} \rightarrow a = \frac{-1}{۶}$$

۴-

$$\lim_{x\to(\frac{-۱}{۲})^+} \frac{۱۶x-[\frac{-۲}{x^۲}]}{۲۴x+[\frac{۳}{x^۲}]}$$

$$x>\frac{-۱}{۲} \to x^۲<\frac{۱}{۴} \rightsquigarrow \frac{۱}{x^۲}>۴ \rightsquigarrow \frac{-۸+۹}{-۱۲^++۱۲}=\frac{۱}{۰^+}=+\infty$$

۵-

$$\lim_{x\to ۲^+} \frac{x^۲-۴}{x^۳-[x^۳]} \xrightarrow{\text{مبهم } \frac{۰}{۰}} \frac{x^۲-۴}{x^۳-۸}=\frac{(\cancel{x-۲})(x+۲)}{(\cancel{x-۲})(x^۲+۲x+۴)}=\frac{۴}{۱۲}=\frac{۱}{۳}$$

۶-

$$\lim_{x\to -۸} \frac{x^۲+۱۰x+۱۶}{۱۲+۶\sqrt[۳]{x}} \rightsquigarrow \frac{(x+۸)(x+۲)}{۶(۲+\sqrt[۳]{x})}\times\frac{(\sqrt[۳]{x^۲}-۲\sqrt[۳]{x}+۴)}{(\sqrt[۳]{x^۲}-۲\sqrt[۳]{x}+۴)}=\frac{(\cancel{x+۸})(x+۲)(\sqrt[۳]{x^۲}-۲\sqrt[۳]{x}+۴)}{۶(\cancel{x+۸})}$$

$$\frac{-\cancel{۶}(\cancel{۴}+\cancel{۴}+۴)}{\cancel{۶}} = \rightsquigarrow -۱۲$$

۷-

$$\lim_{x\to ۰^-} \frac{\sqrt{۲+۳x}-\sqrt{۲-x}}{\sqrt{۱-\cos x}}$$

$$\times\frac{\sqrt{۱+\cos x}}{\sqrt{۱+\cos x}}\times\frac{\sqrt{۲+۳x}+\sqrt{۲-x}}{\sqrt{۲+۳x}+\sqrt{۲-x}} \Rightarrow \lim_{x\to ۰^-}\frac{(۲+۳x-۲+x)(\sqrt{۲})}{(\sqrt{۱-\cos^۲ x})(۲\sqrt{۲})}=\frac{۴x}{۲|\sin x|}$$

$$\rightsquigarrow \frac{۲x}{-\sin x}=-۲$$

-۸

$$\lim k + \cos(\sqrt{a}x) = 0 \longrightarrow k+1 = 0 \Rightarrow k = -1$$

$$\lim_{x\to 0} \frac{-1+\cos(\sqrt{a}\,x)}{-x^2} \xrightarrow{\text{هم‌ارزی}} \lim_{x\to 0} \frac{\frac{-ax^2}{2}}{-x^2} = \frac{a}{2} \longrightarrow \frac{a}{2} = 2$$

$$\boxed{a = 4} \qquad \frac{a}{k} = \frac{4}{-1} = -4$$

-۹

$$\lim_{x\to 3a^+} \frac{2\sqrt{x-3a}}{(\sqrt{x}-\sqrt{3a})(\sqrt{x}+\sqrt{3a})} + \frac{3\sqrt{x}-3\sqrt{3a}}{(\sqrt{x}-\sqrt{3a})(\sqrt{x}+\sqrt{3a})} = \lim_{x\to 3a^+}\left(\frac{2}{\sqrt{4a}} + \frac{3(\sqrt{x}-\sqrt{3a})}{(\sqrt{x}-\sqrt{3a})(\sqrt{x}+\sqrt{3a})}\times\frac{\sqrt{x}+\sqrt{3a}}{\sqrt{x}+\sqrt{3a}}\right)$$

$$\Rightarrow \lim_{x\to 3a^+}\left(\frac{2}{\sqrt{4a}} + \frac{3\sqrt{x-3a}}{(\sqrt{x}+\sqrt{3a})(\sqrt{x}-\sqrt{3a})}\right) \Rightarrow \lim_{x\to 3a^+} \frac{2}{\sqrt{4a}} \longrightarrow \frac{\sqrt{4}}{\sqrt{4a}} = \sqrt{\frac{2}{2a}}$$

-۱۰

$$\lim_{x\to(-1)^-} \frac{1-k[x]}{x^2-1} = \frac{1+2k}{0^+} = -\infty \Rightarrow 1+2k < 0$$

$$\lim_{x\to(-1)^+} \frac{1-k[x]}{x^2-1} \Rightarrow \frac{1+k}{0^-} = -\infty \Rightarrow 1+k > 0$$

$$k < \frac{-1}{2} \quad , \quad k > -1 \Rightarrow -1 < k < \frac{-1}{2}$$

$$\begin{cases} -\pi < k\pi < \frac{-\pi}{2} \\ -1 < \cos k\pi < 0 \end{cases} \Rightarrow$$ طول عرض منفی

ناحیه سوم