پارمیس زندی

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سؤال ۱

$$\lim_{x\to ۲} \frac{۲x-۵}{x^۲+ax+b} = -\infty \qquad a+b=? \qquad \frac{-۱}{0^+}=-\infty$$

$$۴+۲a+b=0 \Rightarrow ۲a+b=-۴$$

$۲x-۵ \Rightarrow ۴-۵=-۱$ ریشه مضاعف 0^+ $(x-۲)^۲ = x^۲+ax+b \Rightarrow x^۲+۴-۴x = x^۲+b+ax$

$$\Rightarrow a=-۴,\ b=۴ \Rightarrow \boxed{a+b=-۴+۴=0}$$

سؤال ۲

$$\lim_{x\to(\sqrt[۳]{۴})} \frac{x}{x^۲+ax+b} = +\infty \qquad \left[\frac{b}{a}\right]=? \qquad \frac{\sqrt[۳]{۴}}{0^+}=+\infty$$

$$(x-\sqrt[۳]{۴})^۲ = x^۲+ax+b \Rightarrow x^۲-۲\sqrt[۳]{۴}x+(\sqrt[۳]{۴})^۲$$

$$\Rightarrow a=-۲\sqrt[۳]{۴} \quad b=(\sqrt[۳]{۴})^۲ \quad \left[\frac{(\sqrt[۳]{۴})^۲}{-۲\sqrt[۳]{۴}}\right] = \left[\frac{-\sqrt[۳]{۴}}{۲}\right] \Rightarrow \boxed{\left[\frac{b}{a}\right]=-۱}$$

سؤال ۳

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$$\lim \frac{[-x]+a}{\left|\frac{\sqrt{۳}}{۲}x+a\right|+a} = -\infty \qquad [a]=?$$

$$x\to\left(\frac{۱}{\sqrt{۳}}\right)^+ = \left(\frac{\sqrt{۳}}{۳}\right)^+ \qquad \frac{-۱+a}{\left|\frac{\sqrt{۳}}{۲}x+a\right|+a} = -\infty$$

$$\frac{[-x]+a}{0} = -\infty$$

$$\left|\frac{\sqrt{۳}}{۲}x+a\right| = -a$$

سؤال ۴

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$$\lim_{x\to(-\frac{۱}{۲})^+} \frac{۱۶x-\left[-\frac{۲}{x^۲}\right]}{۲۴x+\left[\frac{۳}{x^۲}\right]} = ? \Rightarrow \lim_{x\to(-\frac{۱}{۲})^+} \frac{۱۶x+۸}{۲۴x+۱۲} = \frac{۴(۴x+۲)}{\cancel{۴}(۶x+۳)} = \lim_{x\to(-\frac{۱}{۲})^+} \frac{۴x+۲}{۶x+۳}$$

$$= \frac{\cancel{-۲+۲}}{\cancel{-۳+۳}} \xrightarrow{HOP} \frac{\cancel{۴}}{\cancel{۶}} = \boxed{\frac{۲}{۳}}$$

سؤال ۵

$$\lim_{x\to 2^+}\frac{x^2-4}{x^3-[x^3]}=? \Rightarrow \frac{x^2-4}{x^3-8}=\lim_{x\to 2^+}\frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(x^2+2x+4)}=\frac{4}{4+4+4}=\frac{4}{12}$$

$$=\boxed{\frac{1}{3}}$$

سؤال ۶

$$\lim_{x\to -8}\frac{x^2+10x+16}{12+6\sqrt[3]{x}}=\frac{0}{0}\Longrightarrow \text{Hop}\ \lim\frac{2x+10}{\frac{6\times 1}{3\sqrt[3]{64}}}=\frac{-6}{\frac{6}{3\times 4}}$$

$$=\boxed{-12}$$

سؤال ۷

$$\lim_{x\to 0^-}\frac{\sqrt{2+3x}-\sqrt{2-x}}{\sqrt{1-\cos x}}\times\frac{\sqrt{1+\cos x}}{\sqrt{1+\cos x}}\times\frac{\sqrt{2+3x}+\sqrt{2-x}}{\sqrt{2+3x}+\sqrt{2-x}}$$

$$=\lim_{x\to 0^-}\frac{(2+3x-2+x)\times\sqrt{2}}{(\sqrt{1-\cos^2 x})(2\sqrt{2})}=\lim_{x\to 0^-}\frac{4x}{2|\sin x|}=\lim_{x\to 0^-}\frac{2x}{-\sin x}=-2$$

سؤال ۸

$$\lim_{x\to 0}\frac{k+\cos(\sqrt{a}x)}{kx^2}=3 \qquad \frac{a}{k}=?$$

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$$\text{Hop}\ \frac{-\sqrt{a}\sin(\sqrt{a}x)}{2kx}=3\rightarrow \text{Hop}\ \lim_{x\to 0}\frac{-\sqrt{a}\times\sqrt{a}\cos(\sqrt{a}x)}{2k}=\frac{-a\cos(\sqrt{a})}{2k}=3$$

$$-a\cos(\sqrt{a})=6k\rightarrow \frac{-a\cos(\sqrt{a})}{6}=k\rightarrow\frac{a}{k}=\frac{-6}{\cos(\sqrt{a})}$$

$$\lim_{x\to(3a)^+} \frac{2\sqrt{x-3a}+3\sqrt{x}-\sqrt{27a}}{\sqrt{x^2-9a^2}} = \frac{2\sqrt{\cancel{x-3a}}}{\cancel{\sqrt{(x-3a)}}\times\sqrt{(x+3a)}} + \underbrace{\frac{\frac{3(\sqrt{x}-\sqrt{3a})}{3\sqrt{x}-\sqrt{27a}}\ 3\sqrt{3a}}{\sqrt{x-3a}\times\sqrt{x+3a}}}_{*}$$

سوال ۹

$x\longrightarrow(3a)^+$

$a>0$

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$$(*)\ \frac{3(\sqrt{x}-\sqrt{3a})}{\sqrt{x^2-9a^2}}\times\frac{\sqrt{x}+\sqrt{3a}}{\sqrt{x}+\sqrt{3a}} = 3\left(\frac{x-3a}{\sqrt{x^2-9a^2}\times\sqrt{x}+\sqrt{3a}}\right)$$

سوال ۱۰

$$\lim_{x\to(-1)} \frac{1-k[x]}{x^2-1} = -\infty \qquad \frac{\text{عدد}}{0} = -\infty$$

$$\xrightarrow{x\to-1^-}\ \frac{1+2k}{\underbrace{x^2-1}_{0^+}} = -\infty \Rightarrow 1+2k<0 \Rightarrow 2k<-1 \Rightarrow \boxed{k<-\frac{1}{2}}\ \text{(I)}$$

$-1/1$

$$\xrightarrow{x\to-1^+}\ \frac{1+k}{x^2-1} \Rightarrow \frac{1+k}{0^-} = -\infty \Rightarrow 1+k>0 \Rightarrow \boxed{k>-1}\ \text{(II)}$$

-1^+

$(k\pi, \cos k\pi)$

$$\text{(I)},\text{(II)}\ \boxed{-1<k<-\frac{1}{2}} \Rightarrow -\pi<k\pi<-\frac{\pi}{2}$$

ناحیه سوم

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$$\lim_{x\to3a^+} \frac{2\sqrt{\cancel{x-3a}}}{\sqrt{\cancel{x-3a}}\ \sqrt{x+3a}} + \frac{3(\sqrt{x}-\sqrt{3a})}{\sqrt{x-3a}\ \sqrt{x+3a}} = \lim_{x\to3a^+} \frac{2}{\sqrt{6a}} + \frac{3(\sqrt{x}-\sqrt{3a})}{\sqrt{x-3a}\ \sqrt{x+3a}}\times\frac{\sqrt{x}+\sqrt{3a}}{\sqrt{x}+\sqrt{3a}}$$

$$\lim_{x\to3a^+} \frac{2}{\sqrt{6a}} + \frac{3(\cancel{x-3a})}{\sqrt{\cancel{x-3a}}\ \sqrt{x+3a}\ (\sqrt{x}+\sqrt{3a})} = \lim_{x\to3a^+} \left(\frac{2}{\sqrt{6a}} + \frac{3\sqrt{\cancel{x-3a}}}{\sqrt{\cancel{x+3a}}\ (\sqrt{x}+\sqrt{3a})}\right) = \lim_{x\to3a^+} \frac{2}{\sqrt{6a}}+0 \rightsquigarrow \frac{2}{\sqrt{6a}} = \sqrt{\frac{4}{6a}} = \sqrt{\frac{2}{3a}}$$

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حاصل حد هنگامی به سمت بی‌نهایت میل می‌کند که مخرج به صفر میل کند!

$$\lim_{x \to (\frac{\sqrt{3}}{3})^+} \left|\frac{\sqrt{3}}{3}x + a\right| + a = 0 \rightsquigarrow \left|\frac{1}{3} + a\right| + a = 0$$

$$a \geq -\frac{1}{3} \to 2a + \frac{1}{3} = 0 \to \boxed{a = -\frac{1}{6}}$$

$$a < -\frac{1}{3} \to -\frac{1}{3} = 0 \quad \text{غ ق} \; \times$$

$$\rightsquigarrow [a] = -1$$

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$$x > -\frac{1}{2} \to x^2 < \frac{1}{4} \to \frac{1}{x^2} > 4 \to \frac{3}{x^2} > 12 \rightsquigarrow \left[\frac{3}{x^2}\right] = 12$$

$$\hookrightarrow -\frac{2}{x^2} < -8 \rightsquigarrow \left[-\frac{2}{x^2}\right] = -9$$

$$\lim_{x \to (-\frac{1}{2})^+} \frac{16x + 9}{24x + 12} = \frac{-8 + 9}{-12^+ + 12} = \frac{1}{0^+} = \boxed{+\infty}$$

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چون حد مخرج برابر صفر است و حاصل حد عددی حقیقی و غیرصفر است پس حد صورت نیز برابر صفر است!

$$\lim_{x \to 0} k + \cos(\sqrt{a}\,x) = 0 \to k + 1 = 0 \to \boxed{k = -1}$$

$$\lim_{x \to 0} \frac{-1 + \cos\sqrt{a}\,x}{-x^2} \overset{\text{هم‌ارزی}}{\rightsquigarrow} \lim_{x \to 0} \frac{-1 + 1 - \frac{(\sqrt{a}\,x)^2}{2}}{-x^2} = \frac{-ax^2}{2(-x^2)} = \frac{a}{2}$$

$$\frac{a}{2} = 3 \to \boxed{a = 6} \rightsquigarrow \frac{a}{k} = -6$$