

$$\lim_{x \rightarrow r} \frac{rx - a}{x^r + ax + b} = -\infty \quad a+b=? \quad \frac{-1}{0^+} = -\infty$$

مثال (۱)

$$x \rightarrow r$$

$$r + ra + b = 0 \Rightarrow ra + b = -r$$

$$rx - a \sim x - a = -1 \quad \text{فرض کنیم} \quad (x-r)^r = x^r + ax + b \Rightarrow x^r + r - rx = x^r + b + ax$$

$$\Rightarrow a = -r, b = r \Rightarrow \boxed{a+b = -r+r=0}$$

$$\lim_{x \rightarrow r} \frac{x}{x^r + ax + b} = +\infty \quad \left[\frac{b}{a} \right] = ? \quad \frac{\sqrt[r]{r}}{0^+} = +\infty$$

مثال (۲)

$$x \rightarrow (\sqrt[r]{r}) \quad (x - \sqrt[r]{r})^r = x^r + ax + b \Rightarrow x^r - r\sqrt[r]{r}x + (\sqrt[r]{r})^r$$

$$\Rightarrow a = -r\sqrt[r]{r} \quad b = (\sqrt[r]{r})^r \quad \left[\frac{(\sqrt[r]{r})^r}{-r\sqrt[r]{r}} \right] = \left[\frac{-\sqrt[r]{r}}{r} \right] \Rightarrow \boxed{\left[\frac{b}{a} \right] = -1}$$

$$\lim_{x \rightarrow r} \frac{[-x] + a}{\left| \frac{\sqrt[r]{r}}{r} x + a \right| + a} = -\infty \quad [a] = ?$$

مثال (۳)

$$x \rightarrow \left(\frac{1}{\sqrt[r]{r}} \right)^+ = \left(\frac{\sqrt[r]{r}}{r} \right)^+ \quad \frac{-1+a}{\left| \frac{\sqrt[r]{r}}{r} x + a \right| + a} = -\infty$$

$$\frac{[-x] + a}{0} = -\infty$$

$$\left| \frac{\sqrt[r]{r}}{r} x + a \right| = -a$$

$$\lim_{x \rightarrow (-\frac{1}{r})^+} \frac{14x - \left[-\frac{r}{x^r} \right]}{r^r x + \left[\frac{r}{x^r} \right]} = ? \Rightarrow \lim_{x \rightarrow (-\frac{1}{r})^+} \frac{14x + 1}{r^r x + 1^r} = \frac{f(x+r)}{f'(4x+r^r)} = \lim_{x \rightarrow (-\frac{1}{r})^+} \frac{f(x+r)}{4x+r}$$

مثال (۴)

$$x \rightarrow \left(-\frac{1}{r} \right)^+ = \frac{-r + r}{-r + r} \xrightarrow{\text{HOP}} \frac{r}{r} = \boxed{\frac{r}{r}}$$

$$\lim_{x \rightarrow r^+} \frac{x^r - r}{x^r - [x^r]} = ? \Rightarrow \frac{x^r - r}{x^r - r} = \lim_{x \rightarrow r^+} \frac{(x-r)(x+r)}{(x-r)(x^r + rx + r)} = \frac{r}{r + r + r} = \frac{r}{3r} \quad \text{سؤال ٢٥}$$

$$= \boxed{\frac{1}{3}}$$

$$\lim_{x \rightarrow -\infty} \frac{x^3 + 10x + 14}{1x + 4\sqrt[3]{x}} = \frac{\infty}{\infty} \Rightarrow \text{HOP} \lim_{x \rightarrow -\infty} \frac{3x^2 + 10}{1 + \frac{4}{3}\sqrt[3]{x}} = \frac{-\infty}{-\infty}$$

$$= \boxed{-12}$$

$$\lim_{x \rightarrow 0^-} \frac{\sqrt{r+rx} - \sqrt{r-x}}{\sqrt{1-\cos x}} \times \frac{\sqrt{1+\cos x}}{\sqrt{1+\cos x}} \times \frac{\sqrt{r+rx} + \sqrt{r-x}}{\sqrt{r+rx} + \sqrt{r-x}}$$

$$= \lim_{x \rightarrow 0^-} \frac{(r+rx - r+x) \times \sqrt{r}}{(\sqrt{1-\cos^2 x})(r\sqrt{r})} = \lim_{x \rightarrow 0^-} \frac{rx}{r|\sin x|} = \lim_{x \rightarrow 0^-} \frac{rx}{-\sin x} = -r$$

$$\lim_{x \rightarrow \infty} \frac{k + \cos(\sqrt{a}x)}{kx^p} = r \quad \frac{a}{k} = ? \quad \text{سؤال ١٨}$$

$$kx^p = \infty$$

$$\text{HOP} \frac{-\sqrt{a} \sin(\sqrt{a}x)}{rkx} = r \leadsto \text{HOP} \frac{-\sqrt{a} \times \sqrt{a} \cos(\sqrt{a}x)}{rk} = \frac{-a \cos(\sqrt{a})}{rk} = r$$

$$x \rightarrow \infty$$

$$-a \cos(\sqrt{a}) = 4rk \leadsto \frac{-a \cos(\sqrt{a})}{4} = k \leadsto \frac{a}{k} = \frac{-4}{\cos(\sqrt{a})}$$

$$\lim_{x \rightarrow (ra)^+} \frac{r\sqrt{x-ra} + r\sqrt{x} - \sqrt{ra}}{\sqrt{x^2 - 9ar}} = \frac{r\sqrt{x-ra}}{\sqrt{(x-ra) \times (x+ra)}} + \frac{\frac{r(\sqrt{x}-\sqrt{ra})}{r\sqrt{x}-\sqrt{ra}} \cdot r\sqrt{ra}}{\underbrace{\sqrt{ra} \times \sqrt{x+ra}}_{(*)}}$$

⑨ J's

a>.

$$(*) \frac{r(\sqrt{x}-\sqrt{ra})}{\sqrt{x^2-9ar}} \times \frac{\sqrt{x}+\sqrt{ra}}{\sqrt{x}+\sqrt{ra}} = r \left(\frac{x-ra}{\sqrt{x^2-9ar} \times \sqrt{x}+\sqrt{ra}} \right)$$

$$\lim_{x \rightarrow (-1)} \frac{1-K[x]}{x^r-1} = -\infty \quad \frac{1}{0} = -\infty$$

$$\begin{aligned} & \xrightarrow{x \rightarrow -1^-} \frac{1+rk}{\underbrace{x^r-1}_{0^+}} = -\infty \Rightarrow 1+rk < 0 \Rightarrow rk < -1 \Rightarrow \boxed{k < -\frac{1}{r}} \quad \text{① J's} \\ & \xrightarrow{x \rightarrow -1^+} \frac{1+k}{\underbrace{x^r-1}_{-1^+}} = -\infty \Rightarrow \frac{1+k}{0^-} = -\infty \Rightarrow 1+k > 0 \Rightarrow k > -1 \quad \text{②} \end{aligned}$$

(kπ, cos kπ)

$$\text{①, ② } \boxed{-1 < k < -\frac{1}{r}} \Rightarrow \underline{\underline{-\pi < k\pi < -\frac{\pi}{r}}}$$

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