تکلیف شماره ۱ فصل ۲

به نام خدا

[illegible]
دوازدهم دختر B

① $\lim\limits_{x\to 2} \dfrac{2x-5}{x^2+ax+b} = -\infty$

حد صورت: $\lim\limits_{x\to 2} (2x-5) = -1 < 0$ منفی

حد مخرج صفر بوده است $= (x-2)^2 \Rightarrow$ مخرج $= x^2-4x+4$

$\longrightarrow a=-4,\ b=+4$

$a+b=0$

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② $\lim\limits_{x\to \sqrt[3]{4}} \dfrac{x}{x^2+ax+b} = +\infty$

مثبت است ← حد صورت
پس حد مخرج نیز باید ۰ باشد

$x^2+ax+b = (x-\sqrt[3]{4})^2 = x^2 - 2\sqrt[3]{4}x + \sqrt[3]{16} \Longrightarrow a = -2\sqrt[3]{4},\ b = +\sqrt[3]{16}$

$\left[\dfrac{b}{a}\right] = \left[\dfrac{\sqrt[3]{16}}{-2\sqrt[3]{4}}\right] = \left[\dfrac{\sqrt[3]{4}}{-2}\right] = -1$

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③ $\lim\limits_{x\to (\frac{1}{\sqrt{3}})^+} \dfrac{[-x]+a}{\left|\frac{\sqrt{3}}{3}x+a\right|+a} = -\infty$

حد مخرج: $\left|\frac{\sqrt{3}}{3}\times\frac{1}{\sqrt{3}}+a\right|+a=0$

$\left|\frac{1}{3}+a\right|+a=0 \longrightarrow$

$-a = \left|a+\frac{1}{3}\right| \xrightarrow{(\ )^2}$ به توان ۲

$a^2 = a^2 + \frac{2}{3}a + \frac{1}{9} \Rightarrow \frac{2}{3}a = \frac{-1}{9}$

$\boxed{a=\frac{-1}{6}}$

$[a] = \left[\frac{-1}{6}\right] = -1$

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④ $\lim\limits_{x\to (\frac{-1}{2})^+} \dfrac{16x - \left[\frac{-2}{x^2}\right]}{24x+\left[\frac{3}{x^2}\right]} = \lim\limits_{x\to (\frac{-1}{2})^+} \left(\dfrac{16x+9}{24x+12} = \dfrac{1}{24x+12}\right) = +\infty$

$x > \frac{-1}{2} \to x^2 < \frac{1}{4} \to \frac{1}{x^2} > 4 \to \frac{-1}{x^2} < -4 \to \frac{-2}{x^2} < -8 \longrightarrow$ حاصل براکت $= -9$

$x > \frac{-1}{2} \to x^2 < \frac{1}{4} \to \frac{1}{x^2} > 4 \to \frac{3}{x^2} > 12 \longrightarrow$ حاصل براکت در مخرج $= 12$

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$$\lim_{x\to 2^+} \frac{x^2-4}{x^3-[x^3]} = \lim_{x\to 2^+}\left(\frac{x^2-4}{x^3-8} = \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(x^2+2x+4)}\right) = \frac{4}{12} = \frac{1}{3}$$

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$$\lim_{x\to -8}\left(\frac{x^2+10x+16}{12+6\sqrt[3]{x}} \overset{\div \text{مزدوج}}{=} \frac{(x+2)(x+8)}{6(\sqrt[3]{x}+2)} \times \left(\frac{\sqrt[3]{x^2}-2\sqrt[3]{x}+4}{\quad '' \quad '' \quad '' \quad}\right)\right) =$$

$$\lim_{x\to -8} \frac{(x+2)\cancel{(x+8)}(12)}{\cancel{6(x+8)}} = \lim_{x\to -8} 2x+4 = -12$$

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$$\lim_{x\to 0^+} \frac{\sqrt{2+3x}-\sqrt{2-x}}{\sqrt{1-\cos x}} \overset{\div \text{مزدوج}}{=\!=} \lim_{x\to 0^+} \frac{\sqrt{2+3x}-\sqrt{2-x}}{\sqrt{1-\cos x}} \times \frac{\sqrt{2+3x}+\sqrt{2-x}}{\quad '' \quad '' \quad '' \quad} \times \frac{\quad '' \quad '' \quad}{\sqrt{1+\cos x}} =$$

$$\lim_{x\to 0^+}\left(\frac{4x}{\sqrt{2+3x}+\sqrt{2-x}} \times \frac{\sqrt{1+\cos x}}{\sin x}\right) = \frac{4\times\sqrt{2}}{2\sqrt{2}} = 2$$

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$$\lim_{x\to 0} \frac{k+\cos(\sqrt{a}\,x)}{kx^2} = 3 \xrightarrow{\text{hop}} \lim_{x\to 0} \frac{-\sin(\sqrt{a}\,x)}{2kx} = 3 \xrightarrow{\text{hop}}$$

$$\boxed{\frac{0}{k} = ?}$$

$$\lim_{x\to 0} \frac{-\cos(\sqrt{a}\,x)}{2k} = 3 \Rightarrow \frac{-1}{2k} = 3 \longrightarrow 6k = -1 \longrightarrow k = \frac{-1}{6}$$

همچنین:

$$\lim_{x\to 0} \frac{-\sin(\sqrt{a}\,x)}{\frac{-2}{6}x} = 3 \longrightarrow \lim_{x\to 0} \frac{\sin(\sqrt{a}\,x)}{x} = \sqrt{a} = 1 \Rightarrow a = 1$$

$$\frac{a}{k} = \frac{1}{\frac{-1}{6}} = -6$$

$(a>0)$ (9)

$$\lim_{x\to(3a)^+} \frac{2\sqrt{x-3a}+3\sqrt{x}-\sqrt{27a}}{\sqrt{x^2-9a^2}} \overset{Hop}{=} \lim_{x\to 3a^+} \frac{\frac{1}{\sqrt{x-3a}}+\frac{3}{2\sqrt{x}}-0}{\frac{x}{\sqrt{x^2-9a^2}}} =$$

$$\lim_{x\to(3a)^+} \frac{\frac{2\sqrt{x}+3\sqrt{x-3a}}{(2\sqrt{x})(\sqrt{x-3a})}}{\frac{x}{\sqrt{x^2-9a^2}}} = \frac{(2\sqrt{x}+3\sqrt{x-3a})(\cancel{\sqrt{x-3a}})(\sqrt{x+3a})}{2\sqrt{x}\times x\times \cancel{\sqrt{x-3a}}} =$$

$$\lim_{x\to(3a)^+} \frac{(\cancel{2\sqrt{3a}})(\sqrt{6a})}{(\cancel{2\sqrt{3a}})\times 3a} = \frac{\sqrt{6a}}{3a} = \sqrt{\frac{6a}{9a^2}} = \sqrt{\frac{2}{3a}}$$

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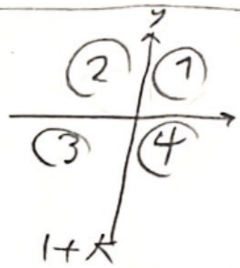
(10)

$$\lim_{x\to -1} \frac{1-k[x]}{x^2-1} = -\infty$$

$(k\pi , \cos k\pi)$ درکدام ناحیه ?



$$\lim_{x\to(-1)^+} \frac{1-k[x]}{x^2-1} = \lim_{x\to(-1)^+} \frac{1+k}{x^2-1} = \frac{1+k}{0^-} = -\infty \to 1+k>0$$

$\boxed{k>-1}$ I

$$\lim_{x\to(-1)^-} \frac{1-k[x]}{x^2-1} = \frac{1+2k}{0^+} = -\infty \to 1+2k<0 \to 2k<-1$$

$\boxed{k<\frac{-1}{2}}$ II

$\text{I}\cap\text{II}: k\in(-1,\frac{-1}{2})$

طول نقاط $= k\pi \xrightarrow{-1<k<\frac{-1}{2}} -\pi<k\pi<\frac{-\pi}{2}$

عرض نقاط $= \cos k\pi \xrightarrow{-1<k<\frac{-1}{2}} -1<\cos k\pi<0$

طول نقاط و عرض نقاط منفی هستند لذا نقاط در ناحیه سوم قرار دارند.