

$$\lim_{x \rightarrow 2} \frac{2x-5}{x^2+ax+b} = -\infty \rightarrow 2x-5 = -1 \rightarrow x^2+ax+b = 0^+ \rightarrow (x-2)^2 = x^2 - 4x + 4$$

$$\begin{cases} a = -4 \\ b = 4 \end{cases} \rightarrow a+b = 4-4 = 0$$

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$$\lim_{x \rightarrow \sqrt[3]{4}} \frac{x}{x^2+ax+b} = +\infty \rightarrow x > 0$$

$$x^2+ax+b = 0^+ \rightarrow (x-\sqrt[3]{4})^2 = x^2 - 2\sqrt[3]{4}x + \sqrt[3]{16}$$

$$\begin{aligned} a &= -2\sqrt[3]{4} \\ b &= \sqrt[3]{16} = 2\sqrt[3]{2} \end{aligned} \rightarrow \frac{b}{a} = \frac{2\sqrt[3]{2}}{-2\sqrt[3]{2}\sqrt[3]{2}} = \left[-\frac{1}{\sqrt[3]{2}} \right] = -1$$

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$$\lim_{x \rightarrow \left(\frac{1}{\sqrt{3}}\right)^+} \frac{[-x]+a}{\left|\frac{\sqrt{3}}{3}x+a\right|+a} \rightarrow [-x]+a = \left[-\frac{1}{\sqrt{3}}\right]+a \rightarrow a-1$$

$$\lim_{x \rightarrow \left(\frac{1}{\sqrt{3}}\right)^+} \frac{a-1}{\frac{1}{3}+2a} = -\infty$$

$$\begin{aligned} \frac{1}{3}+2a &= 0 \\ 2a &= -\frac{1}{3} \\ a &= -\frac{1}{6} \end{aligned}$$

$$[a] = \left[-\frac{1}{6}\right] = -1$$

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$$\lim_{x \rightarrow \left(-\frac{1}{2}\right)^+} \frac{16x - \left[-\frac{2}{x^2}\right]}{24x + \left[\frac{3}{x^2}\right]} = \frac{16x - \left[-\frac{2}{\left(\frac{1}{4}\right)^-}\right]}{24x + \left[\frac{3}{\left(\frac{1}{4}\right)^-}\right]} = \frac{16x - [-8^-]}{24x + [12^+]} = \frac{16x+9}{24x+12}$$

$$\frac{1}{0^+} = +\infty$$

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$$\lim_{x \rightarrow 2^+} \frac{x^2-4}{x^3-8} \stackrel{\text{hop}}{\rightarrow} \frac{2x}{3x^2} = \frac{2}{6} = \frac{1}{3}$$

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$$\lim_{x \rightarrow -8} \frac{x^2 + 10x + 16}{12 + 6\sqrt{x}} \rightarrow \frac{0}{0} \xrightarrow{\text{hop}} \frac{2x+10}{6 \cdot \frac{1}{2\sqrt{x}}} \xrightarrow{x=-8} \frac{-16+10}{6 \cdot \frac{1}{3 \cdot 4}} = \frac{-6}{6 \cdot \frac{1}{12}} = -12$$

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$$\lim_{x \rightarrow 0} \frac{\sqrt{2+3x} - \sqrt{2-x}}{\sqrt{1-\cos x}} \times \frac{\sqrt{2+3x} + \sqrt{2-x}}{\sqrt{2+3x} + \sqrt{2-x}} \times \frac{\sqrt{1+\cos x}}{\sqrt{1+\cos x}} = \frac{2+3x-2+x}{|\sin x|} \times \frac{\sqrt{1+\cos x}}{\sqrt{2+3x} + \sqrt{2-x}}$$

$$= \frac{4x}{-\sin x} \times \frac{\sqrt{2}}{2\sqrt{2}} = \frac{2x}{-\sin x} \xrightarrow{\sin x = x} \frac{2x}{-x} = -2$$

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$$\lim_{x \rightarrow \infty} \frac{k + \cos(\sqrt{a}x)}{kx^2} = 3 \rightarrow \frac{k+1 - \frac{ax^2}{2}}{kx^2} = 3 \rightarrow k+1 - \frac{ax^2}{2} = 3kx^2$$

$$(3k + \frac{a}{2})x^2 - k + 1 = 0 \rightarrow -4(-k+1)(3k + \frac{a}{2})$$

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$$\lim_{x \rightarrow (3a)^+} \frac{2\sqrt{x-3a} + 3\sqrt{x} - \sqrt{27a}}{\sqrt{x^2 - 9a^2}} \xrightarrow{\text{L'H}} \frac{3\sqrt{3a}}{6a}$$

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$$\lim_{x \rightarrow (-1)} \frac{1-k[x]}{x^2-1} = -\infty \xrightarrow{-1^+} \frac{1+k}{0^-} = -\infty \rightarrow 1+k > 0 \rightarrow k > -1$$

$$\xrightarrow{-1^-} \frac{1+2k}{0^+} = -\infty \rightarrow 1+2k < 0 \rightarrow 2k < -1 \rightarrow k < -\frac{1}{2}$$

$$k \in (-1, -\frac{1}{2}) \quad (k\pi, \cos k\pi) \Rightarrow x < 0 \quad y < 0 \rightarrow \text{plot}$$

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