

$$f(a) = \lim_{x \rightarrow a^-} f(x) \rightarrow \lim_{x \rightarrow a^-} \frac{|x^2 + x - 2|}{x(1-x)} = \frac{|a^2 + a - 2|}{a(-1)} = \frac{-1}{a}$$

$$\rightarrow \frac{-1}{a} = b(a - (1)) = -b \rightarrow b = \frac{1}{a} \quad ab = 1 \times \frac{1}{a} = 1$$

$$f(a) = \lim_{x \rightarrow a^+} f(x) = b(x - [-x]) = b(a - (1)) = -b$$

$$x > a \rightarrow -x < -a \rightarrow [-x] = -1$$

$$f(x) = a[x] + a + b[x] + b[a] + b$$

$$= [x](a+b) + a + b + b[a]$$

$$\frac{a[a]}{f(a)} = \frac{a[a]}{a+b+b[a]} = \frac{a[a]}{-a} = -1 \quad b = -a \leftarrow a+b=0$$

(۵) چون f پیوسته است ضرب برکت باید منفی باشد (چون تابع به ازای هر عدد که داخل برکت را صفر کند پیوسته باشد)

$$b=0 \rightarrow f(a) = -1a \quad \frac{a}{f(a)} = \frac{a}{-1a} = -1$$

$$f(x) = \begin{cases} \frac{x^2 + mx + n}{a - x} & x \neq a \\ r & x = a \end{cases} \quad (4)$$

تابع یکتا است، یعنی هیچ a ایست که $x=a$ باشد و $x=a$ نیز باشد
در سوال گفته شده $f(a) = 0$ و $a = r$ و $a = r$

$$S = \frac{-m}{1} = r \rightarrow m = -r, \quad P = \frac{n}{1} = r \rightarrow n = r$$

$$\lim_{x \rightarrow a} \frac{x^2 + mx + n}{a - x} = \frac{r + m}{-1} = -r - m = r \rightarrow -r + r = a = r$$

$$\rightarrow m = -4, n = 1 \rightarrow n - m = 12 \quad \checkmark$$

$$\lim_{x \rightarrow 0^+} f(x) = 1 - a \times 0 + (-1)(r^2 - 1) = 1 - r^2$$

$$\lim_{x \rightarrow 0^-} f(x) = (1 - a)(-1) + 0(r^2 - 1) = a - 1$$

تک نقطه‌ای است

$$-r^2 + 1 = a - 1 \rightarrow r^2 + a - r = 0 \rightarrow a = \frac{r}{r}, a = -1$$

$$a = -1 \rightarrow b \times \sin -\pi = 1 - r = -r \times 0 \quad \checkmark$$

$$a = \frac{r}{r} \rightarrow b \times \sin \frac{r\pi}{r} = -\frac{1}{r} \rightarrow b = \frac{1}{r} \rightarrow \frac{a}{b} = r$$

$$f(x) = \begin{cases} \frac{|x^r + mx - m - 1|}{m|x^r - 1| + |x - 1|} & x \neq 1 \\ \frac{m}{m+r} & x = 1 \end{cases} \quad (\Delta)$$

$$\lim_{x \rightarrow 1} \frac{|x-1| |x+m+1|}{|x-1| (m|x+1|+1)} = \lim_{x \rightarrow 1} \frac{|m+r|}{r m + 1} = f(1) = \frac{m}{m+r}$$

$$m > -r \rightarrow \frac{m+r}{r m + 1} = \frac{m}{m+r}, \quad m = \varepsilon, m = -1$$

$$m < -r \rightarrow \frac{-m-r}{r m + 1} = \frac{m}{m+r} \rightarrow r m^2 + 2m + \varepsilon = 0 \quad \Delta r.$$

$$m = \varepsilon \rightarrow f\left(\frac{1}{\varepsilon}\right) = \left| \frac{1}{\varepsilon} + 1 - a \right|$$

$$f\left(\frac{1}{\varepsilon} - 1\right) + \left| \frac{1}{\varepsilon} - 1 \right| = \frac{9r}{14} \times \frac{14}{14} = \frac{9}{14} \quad \checkmark$$

$$m = -1 \rightarrow f(-1) = \frac{1+1}{-1 \times 0 + 1 - 1} = 1$$

$$f(x) = \begin{cases} \frac{x^r + |x|}{x^r + a|x|} & x \neq 0 \\ \frac{r a - 1}{r a + r} & x = 0 \end{cases} \quad (\Delta)$$

$$\lim_{x \rightarrow 0^+} \frac{x^r + |x|}{x^r - a|x|} = \lim_{x \rightarrow 0^+} \frac{x^r + x}{x^r + a x} \xrightarrow{\text{HOP}} \frac{r x + 1}{r x + a} = \frac{1}{a}$$

$$\lim_{x \rightarrow 0^-} \frac{x^r - x}{x^r - a x} \xrightarrow{\text{HOP}} \frac{r x - 1}{r x - a} = \frac{1}{a}$$

$$\checkmark \quad 0 < \varepsilon \\ a = r, a = \frac{1}{r}$$

$$\frac{1}{a} = \frac{r a - 1}{r a + r} \rightarrow r a^2 - a = r a + r \rightarrow r a^2 - r a - r = 0 \quad \uparrow$$

$$\lim_{x \rightarrow \frac{1}{r}} \frac{x^r + |x|}{x^r + r|x|} = \frac{\frac{1}{r} + \frac{1}{r}}{\frac{1}{r} + 1} = \frac{r}{a}$$

۳- تابع در $x=1$ پیوسته از راست و $x=5$ پیوسته از چپ دارد!

$$f(1) = \tan \frac{\pi}{4} = -1$$

$$\lim_{n \rightarrow 1^+} \frac{|n^2 + n - 2|}{a(1-n)} = \frac{(n+2)(n-1)}{-a(1-n)} = -1 \rightarrow \frac{3}{a} = 1 \rightarrow a = 3$$

$$f(5) = b(5 - [-5]) = 1 \cdot b \quad \lim_{n \rightarrow 5^-} \frac{|25 + 5 - 2|}{3(1-n)} = -\frac{28}{12} = -\frac{7}{3}$$

$$\rightarrow b = -\frac{7}{3} \rightarrow ab = -\frac{7}{3} \times 3 = \boxed{-7}$$

$n=a$ پیوسته $\rightarrow a^2 + a = 0 \rightarrow a = 0$ ✓
 $\rightarrow a = -1 \times$
 $a^2 + a(n-2) + a^2 = 0 \rightarrow -1 - n + 2 + 1 = 0 \rightarrow n = 2$
 $\lim_{n \rightarrow -1} \frac{\sqrt{3}|-n-1|}{|n^2+1|} = \frac{\sqrt{3}}{|1-1+1|} = \frac{\sqrt{3}}{1} = \sqrt{3}$