بسم الله خدا

رهام شکری دوازدهم ریاضی B
تکلیف شماره ۲

(1)

$$f(x)=\begin{cases}\dfrac{\sqrt{4x^2+(m+3)x+\frac{m}{2}}}{|2x^3+(m-3)x^2+a^2|} & x\neq a\\[2ex] \dfrac{2\tan b}{\sqrt{-x}} & x=a\end{cases}$$

$$2a^3+(m-3)a^2+a^2=0 \rightarrow 2a+(m-3)+1=0$$

$$2a=-m+2 \rightarrow m=2-2a \;\star$$

$$\overset{\text{صورت}}{\Longrightarrow} 4a^2+(5-2a)a+1-a=0 \rightarrow 4a^2+4a+1=0 \rightarrow (2a+1)^2=0$$

$$\rightarrow a=\frac{-1}{2}\;\star \rightarrow m=3$$

هم ارزی رادیکال

$$f(x)=\frac{\sqrt{4x^2+4x+\frac{3}{2}}}{2x^3+\frac{1}{4}} \Rightarrow \lim_{x\to\frac{-1}{2}}(f(x))=\frac{\sqrt{6}\,|x+\frac{1}{2}|}{2(x^3+\frac{1}{8})}=\frac{\sqrt{6}}{2(x^2-\frac{x}{2}+\frac{1}{4})}=\frac{2\sqrt{6}}{3}$$

$$\lim_{x\to a}f(x)=f(a) \Rightarrow \frac{2\sqrt{6}}{3}=\frac{2\tan b}{\sqrt{\frac{1}{2}}} \Rightarrow \frac{\cancel{2}\sqrt{6}}{3}\times\frac{\sqrt{2}}{2}=\cancel{2}\tan b \Rightarrow$$

$$\tan b=\frac{\sqrt{3}}{3} \Rightarrow b=\frac{\pi}{6}$$

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(2)

تابع f در نقطه $x=1$ پیوسته است

$$5x^2-ax+b=0$$

$$f(x)=\frac{x^2+ax+b}{x-1}$$

حد پس $x=1$ ریشه عبارت صورت بوده از طرفی حد $f(x)$ موجود است پس صورت تابع f بر $x-1$ بخش پذیر است

$$\begin{cases}5-a+b=0\\ a+b+1=0\end{cases} \rightarrow 2b+6=0 \rightarrow b=-3,\; a=2$$

$$\left[\frac{b-2a}{3}\right]=\left[\frac{-3-4}{3}\right]=-3$$

Year: Month: Day: Subject:

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$$f(1) = \lim_{x \to 1^+} f(x) \Rightarrow \tan \frac{3\pi}{4} = -1$$

$$\lim_{x \to 1^+} \left(\frac{[x^2+x-2]}{a(1-x)} = \frac{x^2-x-2}{a(1-x)} = \frac{(x+2)\cancel{(x-1)}}{-a\cancel{(1-x)}} \right) = -1$$

$$\frac{3}{a} = 1 \longrightarrow a = 3$$

$$f(5) = \lim_{x \to 5^-} f(x) = \lim_{x \to 5^-} \frac{x^2+x-2}{3\cancel{(1-x)}} = \frac{-7}{3}$$

$-(x+2)\cancel{(x-1)}$

$$10b = \frac{-7}{3} \rightarrow b = \frac{-7}{30} \qquad a \times b = 3 \times \frac{-7}{30} = \frac{-7}{10}$$

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$$f(x) = a[x+1] + b[x + [a+1]]$$

$$f(x) = a[x+1] + b([x] + [a+1])$$

$$a[x] + a + b[x] + b[a] + b =$$

$$a = -b$$

$$a + b[a] + \cancel{b} = b[a] = f(a)$$

$$\frac{a[a]}{b[a]} = \frac{a}{b} = -1$$

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$f(x) = b[x^2 - ax] - 2a$

می‌بایست عامل صفر شونده داشته باشیم $b = 0 , 1$ پس

$b = 0 \longrightarrow f(x) = -2a$

$f(a) = -2a$

$\dfrac{a}{f(b)} = \dfrac{a}{-2a} = \dfrac{-1}{2}$

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(۶)

$$\begin{cases} \dfrac{x^2 + mx + n}{a - x} \ ; & x \neq a \\ 2 \ ; & x = a \end{cases}$$

$f(2a) = 0$

$n - m = ?$

$\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) = \lim f(a) = 2$

$\lim_{x \to a} \left(f(x) = \dfrac{x^2 + mx + n}{a - x} \right) \xrightarrow{\text{مشتق}} \dfrac{2x + m}{-1} = 2 \longrightarrow 2x + m = -2$

$\boxed{m = -2 - 2x}$

$\boxed{m = -2 - 2a}$ ✓

$f(2a) = 0 = \dfrac{4a^2 + 2ma + n}{-a} = 0 \longrightarrow 4a^2 + 2a(-2 - 2a) + n = 0$

$\Rightarrow$

$a^2 + am + n = 0 \to a^2 + (-2 - 2a)a + n = 0 \to -a^2 - 2a + n = 0 \longrightarrow -a^2 + 2a = 0 \to a(a - 2) = 0$

$\boxed{-4a + n = 0}$

$a = 0$ غ ق ق

$\boxed{a = 2}$

$a = 2 \longrightarrow m = -6$

$\longrightarrow n = 8$

$n - m = 8 + 6 = 14$ گ۳

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(۷)

$$f(x) = \begin{cases} (1 - a)[x] + (3a^2 - 1)[-x] \ ; & x \notin \mathbb{Z} \\ b \sin\left(\dfrac{\pi}{a}\right) & x \in \mathbb{Z} \end{cases}$$

$\dfrac{a}{b} = ?$

برای مثال اعداد حسابی $x = 1$ و حدود اینجا را بررسی می‌کنیم.

$\lim_{x \to 1^+} f(x) = 1 - a - 6a^2 + 3$

$\lim_{x \to 1^-} f(x) = 0 + -3a^2 + 1$

$\xrightarrow{=} -6a^2 - a + 3 = -3a^2 + 1$

$-3a^2 - a + 2 = 0$

$\div -3 \quad a^2 - a - 6 = 0$

$(a - 3)(a + 2) = 0$

$a = -1$

$a = \dfrac{2}{3}$

$\Rightarrow \lim_{x \to a} f(x) = -2 \ , \ \lim_{x \to a} f(x) = \dfrac{-1}{3}$

$b \xrightarrow{a = -1} b \sin(-\pi) = 0$ غیرقابل قبول

$\xrightarrow{a = \frac{2}{3}} b \sin\left(\dfrac{3\pi}{2}\right) = -b = \dfrac{-1}{3} \longrightarrow b = \dfrac{1}{3} \longrightarrow \dfrac{a}{b} = \dfrac{\frac{2}{3}}{\frac{1}{3}} = 2$

$$f(x) = \begin{cases} \dfrac{|x^2+mx-m-1|}{m|x^2-1|+|x-1|} & ; x \neq 1 \\[2ex] \dfrac{m}{m+2} & ; x = 1 \end{cases}$$

$$\lim_{x \to \frac{1}{m}} f(x) = ?$$

$$\lim_{x \to 1} f(x) = f(1) = \frac{m}{m+2}$$

$$\lim_{x \to 1} f(x) = \frac{\cancel{(x-1)}(x+m+1)}{|\cancel{x-1}|\left(m|x+1|+1\right)} = \frac{m+2}{2m+1} = \frac{m}{m+2}$$

$$(m+2)^2 = 2m^2+m$$

$$m^2+4m+4 = 2m^2+m \longrightarrow m^2-3m-4=0 \longrightarrow (m-4)(m+1)=0 \quad \boxed{m=4} \quad \boxed{m=-1}$$

$$\lim_{\substack{x \to \frac{1}{4} \\ (m=4)}} f(x) = \frac{7}{8}$$

$$\lim_{\substack{x \to -1 \\ (m=-1)}} f(x) = +1$$

$$f(x) = \frac{\sqrt{3}\,|ax+a|}{|x^{3}+(m-2)x+a^{2}|}$$ (۴)

$D_f = R - \{a\}$

$\lim_{x \to a} f(x) =$ موجود

$a^{2}+a=0 \rightarrow a(a+1)=0 \rightarrow a=0$

$\rightarrow a=-1$

$a^{3}+a^{2}+a(m-2)=0 \rightarrow a=0$

$a^{2}+a+(m-2)=0$ $\quad a=-1$

$m=2$

$$\lim_{x \to -1} f(x) = \frac{\sqrt{3}\,|-x-1|}{|x^{3}+1|} = \frac{\sqrt{3}\,|\cancel{x+1}|}{|\cancel{x+1}|\,|x^{2}-x+1|}$$

$$= \frac{\sqrt{3}}{3}$$

$$f(x) = \begin{cases} \dfrac{x^2+|x|}{x^2+a|x|} & ; x \neq 0 \\[2ex] \dfrac{2a-1}{2a+2} & x = 0 \end{cases}$$

$$\lim_{x \to \frac{1}{a}} f(x) = ? \qquad \lim_{x \to 0^+} f(x) = f(0) = \lim_{x \to 0^-} f(x)$$

$$\lim_{x \to 0^+} f(x) = \frac{x^2+x}{x^2+ax} = \frac{x+1}{x+a} = \frac{1}{a} = \frac{2a-1}{2a+2}$$

$$2a+2 = 2a^2 - a \rightarrow 2a^2 - 3a - 2 = 0 \rightarrow a^2 - 3a - 4 = 0$$

$$(a-4)(a+1) = 0 \xrightarrow{\div 2} a = +2, \quad a = \frac{-1}{2}$$

$$\lim_{x \to \frac{1}{a}} f(x) = \begin{cases} \xrightarrow{a = -\frac{1}{2}} \lim_{x \to -2} f(x) = \dfrac{4+2}{4 - \frac{1}{2}(-2)} = \dfrac{6}{5} \\[3ex] \xrightarrow{a = 2} \lim_{x \to \frac{1}{2}} f(x) = \dfrac{\frac{1}{4} + \frac{1}{2}}{\frac{1}{4} + 2(\frac{1}{2})} = \dfrac{\frac{3}{4}}{\frac{5}{4}} = \dfrac{3}{5} \end{cases}$$