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2	$f(x) = \frac{x^2 + ax + b}{x-1} \rightarrow \text{در } x=1 \text{ ناپیوسته است}$ $\lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} \frac{x^2 + ax + b}{x-1} \rightarrow \frac{1^2 + a \cdot 1 + b}{1-1} = \frac{1+a+b}{0}$ $1+a+b=0 \rightarrow a+b=-1$ $5x^2 - ax + b = 0 \xrightarrow{x=1} 5 - a + b = 0 \rightarrow b - a = -5$ $\begin{cases} a+b=-1 \\ b-a=-5 \end{cases} \rightarrow \begin{bmatrix} b-2a \\ 3 \end{bmatrix} \rightarrow \begin{bmatrix} -3-4 \\ 3 \end{bmatrix} = \begin{bmatrix} -7 \\ 3 \end{bmatrix} = -3$ $\frac{a+b=-1}{b-a=-5} \rightarrow 2b=-6 \rightarrow b=-3 \rightarrow a=2$
3	$\tan \frac{3\pi}{4} = \frac{1}{-1} = -1$ $\frac{1}{-1} = \frac{a(1-x)}{a(1-x)} = \frac{-(x+1)(x+2)}{a(1-x)} \rightarrow -\frac{3}{a} = -1 \rightarrow a=3$ $\frac{28}{3(4)} = b(x - [-x]) \rightarrow \frac{7}{3} = b(5+6) \rightarrow 11b = \frac{7}{3} \rightarrow b = \frac{7}{33}$ $ab = \frac{7}{11}$
4	$f(x) = a[x] + a + b[x] + b + b[a]$ تابع پراکنی در $\mathbb{Z}$ ناپیوسته است $f(x) = b[a] \leftarrow a = -b$ $\frac{a[a]}{f(a)} = \frac{a[a]}{-a[a]} = -1 \quad \leftarrow -a[a]$
5	$f(x) = b[x^2 - ax] - 2a \rightarrow \text{چون تابعی که برآوردن نمی‌تواند در } \mathbb{R}$ پایسته باشد $b=0$ $f(x) = -2a \leftarrow b=0$ $\frac{a}{f(b)} = \frac{a}{-2a} = -\frac{1}{2}$

$$f(a) = 2 \quad \lim_{x \rightarrow a} f(x) = \frac{x^2 + mx + n}{a - x} \rightarrow a^2 + ma + n = 0$$

$$\frac{m^2}{9} - \frac{m^2}{3} + n = 0 \rightarrow n = \frac{2m^2}{3}$$

$$\frac{x^2 + mx + \frac{2m^2}{9}}{-(\frac{m}{3} + x)} = \frac{(\frac{m}{3} + x)(\frac{2m}{3} + x)}{-(\frac{m}{3} + x)} \rightarrow \frac{2m}{3} + (-\frac{m}{3}) = -2$$

$$\frac{m}{3} = -2 \rightarrow m = -6, n = 8 \rightarrow n - m = 8 - (-6) = 14$$

$$f(2a) = 4a^2 + 2ma + n = 0$$

$$4a^2 + 2ma + n = a^2 + ma + n$$

$$3a^2 + ma = 0 \rightarrow a(3a + m) = 0$$

$$3a = -m \rightarrow a = -\frac{m}{3}$$

برای اینکه $f(x)$ در R پیوسته باشد باید داشته باشیم

$$1 - a = 3a^2 - 1 \rightarrow 3a^2 + a - 2 = 0$$

$$\rightarrow a = -1 \rightarrow \text{و.ت.}$$

$$\rightarrow a = \frac{2}{3} \rightarrow \text{و.ت.}$$

$$\frac{a}{b} = \frac{\frac{2}{3}}{\frac{1}{3}} = 2$$

$$f(x) = \begin{cases} -\frac{1}{3} & x \notin \mathbb{Z} \\ b \sin(\frac{3\pi}{2}) & x \in \mathbb{Z} \end{cases}$$

$$\hookrightarrow b = \frac{1}{3}$$

$$f(x) = \frac{m}{m+2} \quad \lim_{x \rightarrow 1^+} f(x) = \frac{x^2 + mx - m - 1}{mx^2 + (1-m)x - m + 1} = \frac{2x + m}{2mx + 1 - m} = \frac{2 + m}{m + 1}$$

$$m^2 + m = m^2 + 4m + 4 \rightarrow 3m = -4 \rightarrow m = -\frac{4}{3}$$

$$\lim_{x \rightarrow -\frac{3}{4}} f(x) = \frac{x^2 - \frac{4}{3}x + \frac{1}{3}}{-\frac{4}{3}|x^2 - 1| + |x - 1|} = \frac{x^2 - \frac{4}{3}x + \frac{1}{3}}{\frac{4}{3}x^2 - x - \frac{1}{3}} = \frac{\frac{9}{16} + 1 + \frac{1}{3}}{\frac{3}{4} + \frac{3}{4} - \frac{1}{3}} = \frac{27 + 48 + 16}{48 - 4} = \frac{81}{12} = \frac{27}{4}$$

برای a تعریف نشده: چون a تنها سه خروجی است $(x-a)^3$ می شود

$$x^3 - 3x^2a + 3xa^2 - a^3 = x^3 + (m-2)x + a^2 \rightarrow a = 0$$

$$\lim_{x \rightarrow a} f(x) = \frac{\sqrt{3}(ax + a)}{x^3 + (m-2)x + a^2} = \frac{\sqrt{3}a(a+1)}{a^3 + a^2} = \frac{\sqrt{3}a(a+1)}{a^2(a+2)} = \frac{\sqrt{3}}{2}$$

$$f(x) = \begin{cases} \frac{x^2 + |x|}{x^2 + a|x|} & x \neq 0 \\ \frac{2a-1}{2a+2} & x = 0 \end{cases}$$

$$\lim_{x \rightarrow 0^+} f(x) = \frac{x^2 + x}{x^2 + ax} = \frac{x+1}{x+a} = \frac{2a-1}{2a+2}$$

$$\lim_{x \rightarrow \frac{1}{a}} f(x) \rightarrow \lim_{x \rightarrow -\frac{1}{2}} f(x) = \frac{x^2 - x}{x^2 - \frac{1}{2}x} = \frac{2a^2 - 3a - 2}{2a^2 - 3a - 2} = 0 \rightarrow a = 2 \rightarrow f(0) = \frac{1}{2}$$

$$\lim_{x \rightarrow 0^-} f(x) = \frac{x^2 - x}{x^2 - ax} = \frac{x-1}{x-a} \rightarrow a = 2 \rightarrow -\frac{1}{2}$$

$$\rightarrow a = -\frac{1}{2} \rightarrow -2$$