۱۷,۷۵

۱,۲۵

① $$f'_{+}(1) = \lim_{x\to 1^{+}} \frac{\overbrace{b\sqrt[3]{x^{2}} - b}^{b(\sqrt[3]{x}-1)(\sqrt[3]{x}+1)}}{\underbrace{x-1}_{(\sqrt[3]{x}-1)(\sqrt[3]{x^{2}}+1+\sqrt[3]{x})}} = \frac{b}{3}$$

$$f'_{-}(1) = \lim_{x\to 1^{-}} \frac{\overset{b-1}{a}+\sqrt{x^{2}}-b}{x-1} = \frac{\sqrt{x^{2}}-1}{x-1} = 1$$

$$a+1 = b \Rightarrow a = b-1$$

$$\frac{b}{3} = 1 \Rightarrow b = 3 \quad a = 2 \quad a+b = 5$$

۲) $(f(1) - 3g(1))' = \dfrac{2x-4-3x+4 \cdot 2\sqrt{3x}}{3\sqrt{x^2}} = \dfrac{-2\sqrt{3x}}{3\sqrt{x^2}}$

$\xrightarrow{\text{مسئله}} \dfrac{\sqrt{3}}{3}$

۲

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۳) پیوستگی: $ab + c = \dfrac{1}{a}$

$f'_-(a) = \lim\limits_{x \to a^-} \dfrac{bx + c - \frac{1}{a}}{x-a} = \dfrac{b(x-a)}{x-a} = b$

$f'_+(a) = \lim\limits_{x \to a^+} \dfrac{\frac{1}{x} - \frac{1}{a}}{x-a} = \dfrac{\frac{a-x}{ax}}{x-a} = \dfrac{-1}{ax} = \dfrac{-1}{a^2}$

$ba^2 = -1$

$\underbrace{a^2b}_{-1} + ca = 1 \Rightarrow ac = 2$

۲

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۴) $f'_+(a) = \lim\limits_{x \to a^+} \dfrac{b + (x-a)^m - b}{x-a} = (x-a)^{m-1} = (0)^{m-1}$

$f'_-(a) = \lim\limits_{x \to a^-} \dfrac{b - b}{x-a} = 0$

$\Downarrow$

$m - 1 = 0 \Rightarrow m = 1$

۲

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۵) $g(x) = \dfrac{1}{2x^3} \Rightarrow g'(x) = \dfrac{-6x^2}{4x^6} \Rightarrow g'(-\sqrt[3]{4}) = \dfrac{-3\sqrt[3]{4}}{8}$

$\Rightarrow g(-\sqrt[3]{4}) = \dfrac{-1}{4}$

$f(x) = \dfrac{1}{\sqrt[3]{2x}} \Rightarrow f'(x) = \dfrac{\frac{-2}{3\sqrt[3]{4x^2}}}{\sqrt[3]{4x^2}} = -\dfrac{2}{3}$

$\dfrac{-3\sqrt[3]{4}}{8} \times \dfrac{-2}{3} = \dfrac{\sqrt[3]{4}}{8}$

۱,۵

۶ $f(3) = -3a - 5 \Rightarrow -/5 \quad -a + 3a + 5 = 2$ ۲

$f'(x) = -a \Rightarrow f'(3) = -a \Rightarrow 2a = -3 \Rightarrow a = -1/5$

$\Downarrow$ $1/5 \Rightarrow \frac{f(3)}{f'(3)} = -\frac{1}{3}$

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۷ $(fog)' = f'(g(x)) \times g'(x) = \frac{-2}{\sqrt{8}} \times 4 = -\sqrt{8}$

به ازای ۲

$f(x) = x^2 + 1 \Rightarrow f'(x) = 2x \Rightarrow f'(2) = 4$

$g\left(\frac{3}{\sqrt{8}}\right) = 2$

$g'(x) = \cfrac{-\cfrac{2x}{3\sqrt[3]{(x^2-1)^2}}}{3\sqrt{(x^2-1)^2}} \Rightarrow \frac{-2x}{3} = \frac{-2}{\sqrt{8}}$

۱/۵

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۸ $g'(x) = f'(x+1) + 3f'(3x+10)$

$\Rightarrow g'(-2) = f'(-1) + 3f'(4) = 4f'(-1) = 6$

$f'(-1) = f'(4)$

۲

۹) $\xrightarrow{\text{hop}}$ $\lim\limits_{h\to ۰} \dfrac{\overbrace{-۲f(a-h)}^{-۴}+\overbrace{۳f(a-h)}^{۶}}{-۱} = -۲$ ۱,۵

$f(a)=۲$

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۱۰) $y=\frac{۱}{۲}x+\frac{۱}{۶} \longrightarrow$ شیب خط مماس $=-۲$ ۲

$y'=۲x-۴=-۲ \Rightarrow x=۱ \rightarrow (۱,۲)$

۱

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) \rightsquigarrow \boxed{a+1=b} \qquad f(x)=\begin{cases} a+|x| & x<1 \\ b\sqrt[3]{x^2} & x\geq 1 \end{cases}$$

$$f'_+(1) = f'_-(1) \rightsquigarrow b\times \frac{2x}{3\sqrt[3]{x}} = 1 \longrightarrow b\times\frac{2}{3}=1 \longrightarrow \boxed{b=\frac{3}{2}}$$

$$a=b-1 \longrightarrow a=\frac{1}{2} \rightsquigarrow a+b=\frac{4}{2}=\boxed{2}$$

بخاطر $|x|$ تنها نقطه‌ی مشتق ناپذیر در $x=0$ است (نقطه‌ی گوشه‌ای) پس در ۱ مشتق‌پذیر است.

۵

$$g(-\sqrt[3]{2}) = \frac{1}{(-\sqrt[3]{2})^3 - |-\sqrt[3]{2}|^3} = \frac{1}{-2-2} = -\frac{1}{4}$$

$$f(-\frac{1}{4}) = \frac{1}{\sqrt[3]{-\frac{1}{4}-\frac{1}{4}}} = (-\frac{1}{2})^{-\frac{1}{3}} = -(2)^{-1\times -\frac{1}{3}} = -2^{\frac{1}{3}} = -\sqrt[3]{2} \qquad \longrightarrow fog(x)=x$$

$$g'(x)\times f'(g(x)) = (x)' = 1$$

$$\lim_{h\to 0}\frac{(f(5-h)-2)(f(5-h)-1)}{h(5-h)} = -f'(5)\times\frac{(\overset{2}{f(5)}-1)}{5}$$ ۹

$$f'(5) = \sqrt[3]{x+3} + \frac{x-4}{3\sqrt[3]{(x+3)^2}} \rightsquigarrow -f'(5) = -(\sqrt[3]{8} + \frac{1}{3\times 4}) = 2+\frac{1}{12} = -\frac{25}{12}$$

$$-\frac{25}{12}\times\frac{2-1}{5} = \boxed{-\frac{5}{12}}$$

تمرین مشتق
در $f(5)$ $\to \frac{f(h)-f(5-h)}{h} = f'(5)$

$$(f\circ g)' = g'(\frac{3}{\sqrt{8}})\times f'(g(\frac{3}{\sqrt{8}}))$$ ۷

$$g(\frac{3}{\sqrt{8}}) = \frac{1}{\sqrt[3]{\frac{9}{8}-1}} = 2 \to g'(\frac{3}{\sqrt{8}})\times f'(2)$$

$$g'(\frac{3}{\sqrt{8}}) = -\frac{1}{3}(x^2-1)^{-\frac{4}{3}}\times 2x = \frac{6}{\sqrt{8}}\times -\frac{1}{3}\times(\frac{9}{8}-1)^{-\frac{4}{3}} = -\frac{2}{\sqrt{8}}\times(2^{-3})^{-\frac{4}{3}}$$

$$= -\frac{2^5\times\sqrt{2}}{4} = -8\sqrt{8}$$

$$f(x),\ x=2 \to f(x) = x[\varepsilon]^2+1 = 3x^2+1 \to f'(2) = 32(2) = 96$$

$$\frac{-8\sqrt{2}\times 9\cancel{6}}{-1\cancel{2}8\sqrt{2}} = 4$$