سوال ۱

$$f(x) = \begin{cases} a - x & x < ۱ \\ \sqrt[۳]{x^۲}\, b & x \geq ۱ \end{cases}$$

$$-۱ = \frac{۲}{۳\sqrt[۳]{\cancel{x}_{۱}}}\, b \rightarrow \boxed{-\frac{۳}{۲} = b}$$

$$a - ۱ = b \rightarrow a - ۱ = -\frac{۳}{۲} \rightarrow \boxed{a = -\frac{۱}{۲}}$$

$$a + b = -\frac{۳}{۲} - \frac{۱}{۲} = -۲$$

Scanned with CamScanner

سوال ۲

$f'(1) - 2(1) - 4 = -2 -$

$f(x) = \frac{4-2x}{2\sqrt{x^2}} = f'(x) = \frac{-2(2\sqrt{x^2}) - \frac{2}{2\sqrt{x}}(4-2x)}{(2\sqrt{x^2})^2}$

$f'(x) = \frac{-2\cdot 2\sqrt{x^2} - \frac{2(4-2x)}{2\cdot 2\sqrt{x}}}{x\, 2\sqrt{x}}$

$f'(1) = \frac{-2 - \frac{4}{2}}{1}$

$\boxed{f'(1) = -\frac{10}{2}}$

صورت سو

$g(x) = \frac{|x-2| + \sqrt{2x}}{2\sqrt{x^2}} \qquad \frac{(2-x) + \sqrt{2x}}{3\cdot 2\sqrt{x^2}}$

$\left(-1 + \frac{2}{2\cdot 2\sqrt{2x}}\right)(2\sqrt{x^2})' - \frac{2}{2\cdot 2\sqrt{x}}(-x + 2 + \sqrt{2x})$

$\left(-1 + \frac{2}{2\sqrt{2}}\right)(1) - \frac{2}{2}(+1 + \sqrt{2})$

$-1 + \frac{3}{3\sqrt{2}} - \frac{2}{2} - \frac{2}{2}\sqrt{2}$

$-\frac{5}{2} - \frac{\sqrt{2}}{9}$ ؟

$-\frac{10}{3} - 3\left(-\frac{5}{2} - \frac{\sqrt{2}}{9}\right)$ ؟

سوال ۳

$$\boxed{ab + c = \frac{1}{a}}$$

$$b = -\frac{1}{a^2} \to -a^2 b = 1 \to \boxed{b = -1}$$

$$\to \boxed{a^2 = 1} \qquad \boxed{a = \pm 1}$$

$$-a + c = \frac{1}{a} \to \boxed{c = \frac{a^2+1}{a}}$$

$$ac \to \frac{a^2+1}{\cancel{a}} \times \cancel{a} = a^2 + 1 \to 1 + 1 = 2$$

$- 1 \quad + 1$

سوال

$$g(x) = \frac{1}{x^3-(-x^3)} \to \frac{1}{2x^3}$$

$x \to -2\sqrt{2}$

قرارقعله - x

$$g'(x) = \frac{-6x^2}{(2x^3)^2} = -\frac{3}{2x^4}$$

$$g'(-2\sqrt{2}) = \frac{-3}{2(-2\sqrt{2})^4} = \frac{-3}{22\sqrt{16}}$$

$$g(-2\sqrt{2}) = \frac{1}{2(-2\sqrt{2})^3} = -\frac{1}{4}$$

$$f(x) = \frac{1}{\sqrt[3]{x-(-x)}} = \frac{1}{\sqrt[3]{2x}}$$

$$f'(x) = \frac{\frac{-2}{3\sqrt[3]{4x^2}}}{\sqrt[3]{4x^2}} = \frac{-2}{3\sqrt[3]{16x^4}}$$

$$f'(g(-2\sqrt{2})) = f'(-\tfrac{1}{4}) = -\frac{2}{3\sqrt[3]{16 \times \frac{1}{256}}} = \frac{-2\sqrt[3]{16}}{3}$$

$$-\frac{3}{22\sqrt{16}} \times \frac{-2\sqrt[3]{16}}{3} = 1$$

سوال۴

$y = -ax - ۵$

$f'(۲) - f(۲) = ۲$

$f'(۲) = -a$

$f(۲) = -۲a - ۵$

$f'(۲) - f(۲) = ۲ \qquad -a - (-۲a - ۵) = ۲ \rightarrow a = -\frac{۳}{۲}$

$\frac{f(۲)}{f'(۲)} = \frac{-۲a - ۵}{-a} = -\frac{\frac{۱}{۲}}{\frac{۳}{۲}} = -\frac{۱}{۳}$

$$(f \circ g)'\left(\frac{3}{\sqrt{8}}\right) = g'\left(\frac{3}{\sqrt{8}}\right) f'\left(g\left(\frac{3}{\sqrt{8}}\right)\right)$$

$$g\left(\frac{3}{\sqrt{8}}\right) = \frac{1}{\sqrt[3]{\left(\frac{3}{\sqrt{8}}\right)^2 - 1}} = \frac{1}{\sqrt[3]{\frac{9}{8} - 1}} = 2$$

$$g'\left(\frac{3}{\sqrt{8}}\right) f'(2)$$

$$g'(x) = -\frac{1}{3}\, 2x\,(x^2-1)^{-\frac{4}{3}}$$

$$= -\frac{2}{3}\, \cancel{x}\,(x^2-1)^{-\frac{4}{3}} = -8\sqrt{2}$$

$$\frac{3}{\sqrt{8}} \quad \frac{9}{8}$$

$$f(x) = (4x)^2 + 1 = 16x^2 + 1 \Rightarrow f'(x) = 32x$$

$$f'(2) = 64 \qquad (f \circ g)'\left(\frac{3}{\sqrt{8}}\right) = -8\sqrt{2}\,(64)$$

$$4(-128\sqrt{2})$$

$$\frac{4(-128\sqrt{2})}{128\sqrt{2}} = -4$$

$$= \boxed{\cancel{-512\sqrt{2}}}$$

سوال ۸

$$g'(x) = f'(x+1) + ۳ f'(۳x+۱۰)$$

$$g'(-۳) = f'(-۱) + ۳ f'(۱) = f'(-۱)$$

طبق دوره تناوب

$$g'(-۳) = ۴ f'(-۱)$$

$$g'(-۳) = \cancel{۴}^{۲} \times \frac{۳}{\cancel{۲}} = ۶$$

سوال ۹

$$\lim_{h\to 0} \frac{f^2(5-h) - 4f(5-h) + 4}{h(5-h)}$$

$$\lim_{h\to 0} \frac{2(-1) f(5-h) f'(5-h) - 4(-1) f'(5-h)}{5-2h}$$

هوپیتال

$$= \frac{-2 f(5) f'(5) + 4 f'(5)}{5} \qquad f(5)=2$$

$$f'(x) = (1)(\sqrt[3]{x+3}) + \frac{1}{3\sqrt[3]{(x+3)^2}}(x-4)$$

$$f'(5) = \sqrt[3]{8} + \frac{1}{3\sqrt[3]{64}} \times (5-4) = 2 + \frac{1}{12} = \frac{25}{12}$$

$$\frac{-2 \times \frac{25}{12} \times 2 + 4 \times \frac{25}{12}}{5} = \frac{-\frac{25}{12}}{5} = \boxed{-\frac{5}{12}}$$

سوال ۱۰

$$4y - 2x = 1$$

$$4y = 2x + 1 \qquad \text{قرینه و معکوس} \rightarrow \text{خطوط عمودند}$$

$$y = \frac{2}{4}x + \frac{1}{4}$$

$$\frac{1}{2} \rightarrow -2$$

مشتق تابع $x^2 - 4x + 5$:

$$2x - 4 = -2 \qquad 2x = 2 \rightarrow \boxed{x = 1}$$

کاظم رستمیان

۱۲ اسفند