حل تمرین‌ها

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(1)

$$f(x)=\begin{cases} a+\sqrt{x^2} & x<1 \\ b\sqrt[3]{x^2} & x\geq 1 \end{cases}$$

$$a+\sqrt{x^2}=b\sqrt[3]{x^2}\xrightarrow{x=1} a+1=b$$

$$f'_+(1)=f'_-(1)\Longrightarrow 1=\frac{2}{3}bx^{\frac{-1}{3}}\Rightarrow 1=\frac{2}{3}b\Rightarrow b=\frac{3}{2}\quad a=\frac{1}{2}$$

$$a+b=2$$

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(2)

$$f(x)=\frac{|2x-4|}{\sqrt[3]{x^2}}\qquad g(x)=\frac{\sqrt{x^2-4x+4}+\sqrt{3x}}{\sqrt[3]{x^2}}$$

$$\downarrow\qquad\qquad\downarrow$$

$$f(x)=\frac{4-2x}{\sqrt[3]{x^2}}\qquad g(x)=\frac{|x-2|+\sqrt{3x}}{\sqrt[3]{x^2}}=\frac{2-x+\sqrt{3x}}{\sqrt[3]{x^2}}$$

$$f'(x)=\frac{(-2)\sqrt[3]{x^2}-(\frac{2}{3}x^{\frac{-1}{3}})(4-2x)}{(\sqrt[3]{x^2})^2}\rightarrow f'(1)=\frac{-2\times1-\frac{2}{3}\times2}{1}=\frac{-2-\frac{4}{3}}{1}=\frac{-10}{3}$$

$$g'(x)=\frac{(-1+\frac{3}{2\sqrt{3x}})(\sqrt[3]{x^2})-(\frac{2}{3}x^{\frac{-1}{3}})(2-x+\sqrt{3x})}{(\sqrt[3]{x^2})^2}=-1+\frac{3}{2\sqrt{3}}-\frac{2}{3}-\frac{2\sqrt{3}}{3}=$$

$$-1+\frac{\sqrt{3}}{2}-\frac{2}{3}-\frac{2\sqrt{3}}{3}\Rightarrow\frac{-5}{3}\;\frac{-\sqrt{3}}{6}=g'(1)$$

$$f'(1)-2g'(1)=\frac{-10}{3}+\frac{10}{3}+\frac{\sqrt{3}}{3}=\frac{\sqrt{3}}{3}$$

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$$f(x)=\begin{cases} bx+c & x<a \\ \frac{1}{x} & x\geq a \end{cases} \qquad f'_+(a)=f'_-(a) \qquad f_+(a)=f_-(a)$$

$$\frac{1}{a}=ba+c \qquad b=\frac{-1}{a^۲} \Rightarrow ba^۲=-1 \; *$$

$$ba^۲+ca=1 \xrightarrow{*} -1+ca=1 \Rightarrow ca=۲$$

(۴)

$$f(x)=\begin{cases} b & x<a \\ b+(x-a)^m & x\geq a \end{cases} \qquad f'_-(a)\neq f'_+(a) \Rightarrow$$

$$0\neq m(x-a)^{m-1}\cdot(x-a)$$

در هر صورت برابر صفر می‌شود به غیر از زمانی که $m=1$

(۵)

$$f(x)=\frac{1}{\sqrt[۳]{x-|x|}} \qquad g(x)=\frac{1}{x^۳-|x|^۳}=\frac{1}{۲x^۳} \Rightarrow \frac{۳}{۲x^٤}=g'(x)$$

$$fog \rightarrow \frac{1}{\sqrt[۳]{x^۳-|x^۳|-|x^۳|+x^۳}}=\frac{1}{\sqrt[۳]{۲(x^۳-|x^۳|)}}=\frac{1}{x\sqrt[۳]{۴}} \qquad fog'=\frac{\sqrt[۳]{٤}}{٤x^۲}$$

$$g'(-\sqrt[۳]{۲})\times fog'(-\sqrt[۳]{۲})=\frac{۳}{۴\sqrt[۳]{۲}}\times\frac{\sqrt[۳]{٤}}{۸\sqrt[۳]{۲}}=$$

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$f'(۳) = -a \quad f(۳) = -۳a - ۵$

$-a + ۳a + ۵ = ۲ \rightarrow ۲a = -۳ \rightarrow a = \frac{-۳}{۲} \rightarrow f(۳) = \frac{۹}{۲} - ۵ = \frac{-۱}{۲}$

$\frac{f(۳)}{f'(۳)} = \frac{-۱}{۲} \times \frac{-۲}{۳} = \frac{۱}{۳}$

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$$(fog)' \Rightarrow f'(g)\times g'(x) \Rightarrow \underbrace{\frac{-1}{3}(x^2-1)^{\frac{-4}{3}}}_{g'(x)} \times \underbrace{f'(2)}_{f'(g)} =$$

$$f'(g(\tfrac{3}{2})) = f'(2)$$

$$(x^2-1)^{\frac{-4}{3}} \times \frac{-1}{3} \times 32x \rightarrow \frac{16 \times \frac{-1}{3} \times 64}{-128\sqrt{2}} = \frac{4\sqrt{2}}{3}$$

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$$g(x) = f(x+1) + f(3x+10) \rightarrow g'(-3) = f'(-1) + 3f'(4) \rightarrow$$

$$g'(-3) = \frac{3}{2} + \frac{9}{2} = \frac{12}{2} = 6$$

$$f'(-1) = \frac{3}{2}$$

$\xrightarrow{\text{دوره تناوب ۵}} f'(4) = \frac{3}{2}$

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$$f(x) = (x-4)\sqrt[3]{x+3} \qquad f(5) = 2 \qquad -f'(5)\, f(5)$$

$$\lim_{h\to 0} \frac{f^2(5-h) - 3f(5-h) + 2}{h(5-h)} \Rightarrow \frac{(f(5-h)-1)(f(5-h)-2)}{h(5-h)}$$

$$\frac{-(2-1)f'(5)}{5} = \frac{-f'(5)}{5}$$

$$f'(x) = (\sqrt[3]{x+3}) + (x-4)\left(\frac{1}{3}(x+3)^{\frac{-2}{3}}\right) = f'(5) = 2 + \frac{1}{12} = \frac{25}{12}$$

$$\frac{25}{12} \times \frac{-1}{5} = \frac{-5}{12}$$

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$4y - 2x = 1 \rightarrow 4y = 2x + 1 \rightarrow y = \frac{1}{2}x + \frac{1}{4} \rightarrow$

خط عمود بر $\Rightarrow -2x + b = y \rightarrow$ شیب

$y = x^2 - 4x + 5 \rightarrow 2x - 4 = y' \rightarrow 2x - 4 = -2 \rightarrow x = 1$

$y = x^2 - 4x + 5 \xrightarrow{x=1} y = 2 \rightarrow (1,2)$