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حسین متین پور   کلیف ۲۲   رحمت B

۳) $\begin{cases} bx+c & x<a \\ \frac{1}{x} & x\geqslant a \end{cases}$ b $\frac{-1}{x^2}$

$ab+c=\frac{1}{a} \Rightarrow a^2b+ac=1$ $ac=2$

$b=\frac{-1}{a^2} \rightsquigarrow a^2b=-1$

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٤) $m>0 \xrightarrow{f'(x)} 0=m(0)$

پیوسته!

$m=0 \xrightarrow{f'(x)} 0$

جمع متلی m

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۵) $g'(-\sqrt[3]{2})f'(g(-\sqrt[3]{2}))=(fog)' \Rightarrow (x)'=1 \rightarrow (fog(-\sqrt[3]{2}))'=1$

$x<0 \rightsquigarrow g(x)=\frac{1}{x^3+x^3}$ $(fog)=\frac{1}{\sqrt[3]{\frac{1}{2x^3}+\frac{1}{2x^3}}}=\frac{1}{\sqrt[3]{\frac{1}{x^3}}}=x$

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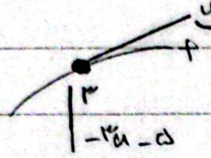
6) 

$y=-ax-5 \xrightarrow[x=3]{f(3)} y=-3a-5$ ٢

y شیب $\Rightarrow -a$ $\qquad f'(3)-f(3)=2$

$f'(3)$ $\qquad -a+3a+5=2$ ارفاق

$\frac{f(3)}{f'(3)} = \frac{-3\times\frac{-3}{2}-5}{-(-\frac{3}{2})} = \frac{\frac{9}{2}-5}{\frac{3}{2}} = \frac{\frac{9-10}{2}}{\frac{3}{2}} = \frac{1}{3}$ $\qquad 2a=-3 \quad a=\frac{-3}{2}$

$-\frac{1}{2}\times\frac{2}{3}=-\frac{3}{2}$

7) $(f\circ g)' = f'(g(x))\times g'(x)$ $\qquad g\left(\frac{3}{\sqrt{8}}\right) = \frac{1}{\sqrt[3]{\frac{9}{8}-1}} = 2$

$g'\left(\frac{3}{\sqrt{8}}\right) = \frac{-\frac{-2x}{3\sqrt[3]{(x^2-1)^2}}}{\sqrt[3]{(x^2-1)^2}} = \frac{-2x}{3\sqrt[3]{(x^2-1)^4}} = \frac{-2\times\frac{3}{\sqrt{8}}}{3\sqrt[3]{(\frac{9}{8}-1)^4}} = \frac{-16}{\sqrt{2}} = -8\sqrt{2}$ ٢

$\to f'(g(x))\times g'(x)$

$f(x) = \left(x\left[4+\frac{1}{2}\right]\right)^2+1 = 16x^2+1 \to f'(2) = 32\times 2 = 64$ $\qquad \frac{\cancel{64}\times -8\sqrt{\cancel{2}}}{-\cancel{128}\sqrt{\cancel{2}}} = 4$

8) $g'(x) = f'(x+1) + 3f'(3x+10) \xrightarrow[x=-2]{} f'(-1)+3f'(4)$ → با توجه به مورد قبل

$\frac{3}{2} + 3\times\frac{3}{2}$ $\qquad -1+5=4$

$= \frac{12}{2} = 6$ ٢ $\qquad f'(-1)=f'(4)=\frac{3}{2}$

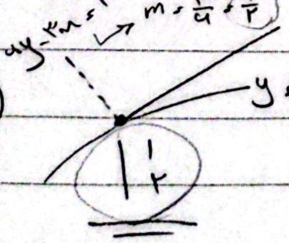
9) $\lim_{h\to 0} \frac{-2f(5-h)f'(5-h)+3f'(5-h)}{5} = \frac{-2f(5)\cdot f'(5)+3f'(5)}{5} = \frac{-\cancel{2}\times 2\times\frac{25}{\cancel{12}}+3\times\frac{25}{\cancel{12}}}{5}$

$5-2h$

$5h-2h$ $\qquad$ ٢ $\qquad \frac{\frac{-25}{12}}{5} = \frac{-5}{12}$

$f(5)=1\sqrt[3]{5+3}=2$ $\qquad f'(5) = \sqrt[3]{x+3} + \frac{(x-5)}{3\sqrt[3]{(x+3)^2}} = 2+\frac{1}{12} = \frac{25}{12}$

$\frac{24+1}{12}$

10) 

$y-3x=1 \to x=\frac{3}{9}=\frac{1}{3}$ $\to x=-2$

$y=x^2-6x+5$ $\to 2x-6=-2$

$2x=2$ ٢

$x=1$ $\quad y=1-6+5=2$

$\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^+} f(x) \rightsquigarrow \boxed{a+1=b}$ $\qquad f(x) = \begin{cases} a+|x| & x<1 \\ b\sqrt[3]{x^2} & x\geq 1 \end{cases}$ ۱

$f'_+(1) = f'_-(1) \rightsquigarrow b\times\frac{2x}{3\sqrt[3]{x}} = 1 \to b\times\frac{2}{3} = 1 \to \boxed{b=\frac{3}{2}}$

$a = b-1 \to a = \frac{1}{2} \rightsquigarrow a+b = \frac{4}{2} = \boxed{2}$

بخاطر $|x|$ تنها نقطه‌ی مشتق ناپذیر در $x=0$ است (نقطه‌ی گوشه‌ای) پس در ۱ مشتق‌پذیر است.

$f(x) = \frac{4-2x}{\sqrt[3]{x^2}} \to f'(1) = \frac{-2(\sqrt[3]{1}) - \frac{2}{3}(1)^{-\frac{1}{3}}\overset{2}{(\cancel{4}-2)}}{\sqrt[3]{1}} = -\frac{10}{3}$ ۲

$g(x) = \frac{|x-2| + \sqrt{3x}}{\sqrt[3]{x^2}} \to g'(1) = \frac{(-1+\frac{3}{2\sqrt{3}})(1) - \frac{2}{3\sqrt[3]{1}}(1+\sqrt{3})}{\sqrt[3]{1^4}}$

$g'(1) = -1 + \frac{3\sqrt{3}}{6} - \frac{2}{3} - \frac{2\sqrt{3}}{3} = -\frac{5}{3} - \frac{\sqrt{3}}{6}$

$f'(1)\ 3g'(1) = -\frac{15}{3} + \frac{10}{3} + \frac{2\sqrt{3}}{6} = \boxed{\frac{\sqrt{3}}{3}}$

نقطه‌ی گوشه‌ای $\to f_+(a) = f_-(a)$ ۳

$m=0 \to$ مشتق هردو طرف برابر است ✗

$m=1 \to f'_-(a) = 0\ ,\ f'_+(a) = 1$ ✓ $\longrightarrow \boxed{m=1}$

$\geq 2 \to f'_-(a) = 0\ ,\ f'_+(a) = m(\underbrace{x-a}_{a-a=0})^{m-1}$ ✗