باران تسلیمی

۱_ تابع در $x=0$ مشتق ناپذیر است.

$$f(x) = \begin{cases} a + \sqrt{x^2} & ; \quad x < 1 \\ b\sqrt[3]{x^2} & ; \quad x \geq 1 \end{cases}$$

$$f(x)_{x\to 1^+} = f(x)_{x\to 1^-} \rightsquigarrow b\sqrt[3]{x^2} = a + \sqrt{x^2}$$

$$b = a + 1 \to b - a = 1$$

$$f'(1)_{x\to 1^+} = f'(1)_{x\to 1^-} \rightsquigarrow 1 \times 1 = b \times x^{\frac{2}{3}} \to 1 = b \times \frac{2}{3} \times x^{-\frac{1}{3}}$$

$$1 = \frac{2}{3} b \to b = \frac{3}{2} \qquad \boxed{a + b = 2}$$

$$a = \frac{1}{2}$$

$$f(x) = \frac{|2x - 4|}{\sqrt[3]{x^2}} \xrightarrow{x\to 1} \frac{4 - 2x}{\underbrace{\sqrt[3]{x^2}}_{x^{\frac{2}{3}}}} \xrightarrow{f'(1)} \frac{-2(x^{\frac{2}{3}}) - \frac{2}{3} x^{-\frac{1}{3}}(4 - 2x)}{\sqrt[3]{x^4}} = \frac{-10}{3} \quad -2$$

$$g(x) = \frac{|x - 2| + \sqrt{3x}}{\sqrt[3]{x^2}} \xrightarrow{x\to 1} \frac{2 - x + \sqrt{3}\, x^{\frac{1}{2}}}{x^{\frac{2}{3}}} \xrightarrow{g'(1)} \frac{\left(-1 + \frac{\sqrt{3}}{2} x^{-\frac{1}{2}}\right)\left(x^{\frac{2}{3}}\right) - \left(\frac{2}{3} x^{-\frac{1}{3}}\right)\left(2 - x + \sqrt{3}\, x^{\frac{1}{2}}\right)}{\sqrt[3]{x^4}}$$

$$\Rightarrow \frac{-10 - \sqrt{3}}{6} \longrightarrow f'(1) - 2g'(1) = \frac{-10}{3} - 2\left(\frac{-10 - \sqrt{3}}{6}\right) = \frac{\sqrt{3}}{3}$$

۳-

$$f(x) = \begin{cases} bx + c \quad ; \quad x < a \\ \dfrac{1}{x} \quad ; \quad x \geqslant a \end{cases} \qquad a \neq 0 \qquad ac\,?$$

$$\underset{x \to a^+}{f(x)} = \underset{x \to a^-}{f(x)} \rightarrow ab + c = \frac{1}{a} \longrightarrow a^۲ b + ac = ۱$$

$$\underset{x \to a^+}{f'(a)} = \underset{x \to a^-}{f'(a)} \rightarrow b = \frac{-۱}{a^۲} \longrightarrow a^۲ b = -۱$$

$$-۱ + ac = ۱ \longrightarrow \boxed{ac = ۲}$$

۴-

$$f(x) = \begin{cases} b \quad ; \quad x < a \\ b + (x-a)^m \quad ; \quad x \geqslant a \end{cases}$$

به ازای چند مقدار صحیح نامنفی m، (a,b) یک نقطه گوشه‌ای برای تابع است؟

$$\underset{x \to a^+}{f'(a)} \neq \underset{x \to a^-}{f'(a)}$$

$$f'_-(a) = ۰ \qquad f'_+(a) \neq ۰ \longrightarrow m(x-a)^{m-۱} \neq ۰ \longrightarrow \boxed{m = ۱}$$

* به ازای یک مقدار از m

$x = -\sqrt[۳]{۲}$

$f(x) = \frac{1}{\sqrt[۳]{۲x}} \qquad g(x) = \frac{1}{۲x^{۳}}$ — ۵

$g'(-\sqrt[۳]{۲}) \times f'(g(-\sqrt[۳]{۲})) = (fog)'$

$fog(-\sqrt[۳]{۲}) = \frac{1}{\sqrt[۳]{\frac{1}{x^{۳}}}} = x \rightarrow (fog)' = x' = 1$

$$\cancel{-\frac{30}{9}} \quad \frac{4(14+5\sqrt{3})}{9}$$

۶-

در $x=3$ بر f مماس

$$f'(3) - f(3) = 2$$

$f(3)$ چند برابر $f'(3)$ است.

$$y = -ax - 5$$

$$-a = f'(3) = \text{شیب}$$

$$-\begin{cases} -3f'(3) - 5 = f(3) \\ f'(3) - 2 = f(3) \end{cases}$$

$$4f'(3) + 3 = 0$$

$$f'(3) = \frac{-3}{4}$$

$$\frac{-3}{4} - f(3) = 2 \longrightarrow f(3) = \frac{-11}{4}$$

$$\frac{f(3)}{f'(3)} = \frac{\frac{-11}{4}}{\frac{-3}{4}} = \frac{11}{3}$$ برابر

$$f(x) = \left(x\left[x^2+\frac{1}{2}\right]\right)^2 + 1 \qquad g(x) = \frac{1}{\sqrt[3]{x^2-1}}$$ -۷

* مقدار مشتق fog در $x=\frac{3}{\sqrt{8}}$ چند برابر $-128\sqrt{2}$ است؟

$$(fog)' = g' \times f'(g)$$

$$g'\left(\frac{3}{\sqrt{8}}\right) = \frac{\frac{-2x}{3\sqrt[3]{(x^2-1)^2}}}{\sqrt[3]{(x^2-1)^2}} = \frac{\left(\frac{-\frac{3}{\sqrt{2}}}{\frac{3}{4}}\right) = \frac{-4}{\sqrt{2}}}{\frac{1}{\frac{1}{4}}} = \frac{-16}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{-16\sqrt{2}}{2} = -8\sqrt{2}$$

$$g(x) = 2$$

$$f'(2) = \left(\underbrace{x\left[\frac{1}{2}\right]}_{0} + x^3\right)^2 + 1 \longrightarrow x^6 + 1 \longrightarrow f'(x) = 6x^5 \longrightarrow f'(2) = 6 \times 2^5$$

$$\longrightarrow \frac{-8\cancel{\sqrt{2}} \times 6 \times 2^5}{\underset{16}{\cancel{-128}}\cancel{\sqrt{2}}} = \boxed{\text{۱۲ برابر}}$$

۸-

$g(x) = f(x+1) + f(3x+10)$, $f'(-1) = \frac{3}{2}$

حاصل $g'(-2) = ?$

$g'(x) = f'(x+1) + 3f'(3x+10)$

$g'(-2) = f'(-2+1) + 3f'(-6+10) = f'(-1) + 3f'(4)$

$f'(4) = f'(-1) \rightarrow g'(-2) = 4f'(-1) \rightarrow 4 \times \frac{3}{2} = 6$

۹-

$-\frac{1}{5} \lim_{t \to 0} \frac{f(5+t) - f(5)}{t} = -\frac{1}{5} f'(5)$

$f'(5) = \sqrt[3]{8} + \frac{1}{3\sqrt[3]{64}} = \frac{25}{12}$

$-\frac{1}{5} f'(5) = -\frac{1}{5} \times \frac{25}{12} = -\frac{5}{12}$

۱۰-

$6y = 3x + 1 \rightarrow y = \frac{1}{2}x + \frac{1}{6}$ شیب $= \frac{1}{2}$

'شیب $= -2$

$y' = 2x - 4 = -2$

$x = 1 \rightarrow y = 1 - 4 + 5 = 2$ $\left| \begin{array}{l} x = 1 \\ y = 2 \end{array} \right.$