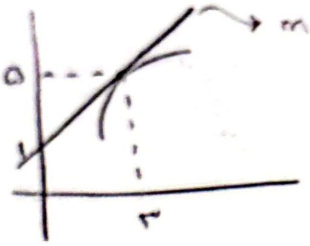
نام و نام خانوادگی .... کلاس .... شماره ....

نام و نام خانوادگی؟ صفر



$\to m = \frac{۶}{۳} = f'(۳)$ ✓ ۲ ۱

$|^{-۱}_{۱} \quad |^{۲}_{۲}$ $\quad y-۲ = \frac{۱}{۳}(x-۲)$ $\quad \sqrt{ax-۱} = \frac{x+۴}{۳}$

$m = \frac{۱}{۳}$ $\quad y = \frac{۱}{۳}x + \frac{۴}{۳}$ $\quad \frac{a}{۲\sqrt{ax-۱}} = \frac{۱}{۳}$ $\quad a = ۲$ ✓ $\quad a = -\frac{۲}{۹}$ غق ۲

شرط مماس $\quad f(a) = g(a)$, $\quad f'(a) = g'(a)$

$f(۲) = \sqrt{۲\times۵-۱} = ۳$ ✓ ۲

$a \left| \begin{array}{l} ۱ \\ \frac{۲+m}{۴} \end{array} \right.$ $\quad y' = \frac{-۳x}{۴} - \frac{n}{۴}$

$\xrightarrow{\text{مشتق}} \frac{۲+m}{۴} = \frac{۳}{۴} - \frac{n}{۴}$ ۱٫۵ ۳

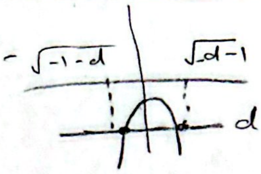
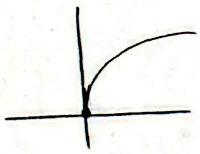
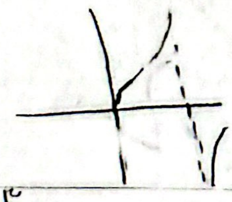
$\frac{m+n}{۴} = \frac{۱}{۴} \quad \underline{m+n = ۱}$

$f = \frac{(۳-\sin x)(۹+\sin^۲ x+۳\sin x)}{(۳-\sin x)(۳+\sin x)} = \frac{۳\sin x+۹}{\sin x+۳} + \frac{\sin^۲ x}{\sin x+۳} = ۳ + \frac{\sin^۲ x}{\sin x+۳}$

$f'(x) = \frac{۲\cos x \sin x(\sin x+۳) - \cos x \sin x \sin x}{(\sin x+۳)^۲} = \frac{\sin x\cos x(\sin x+۳-\frac{۱}{۲}\sin x)}{(\;)^۲}$ $\quad \sin(\frac{۵\pi}{۳}) = -\frac{\sqrt{۳}}{۲}$ ۴

$f'(x) = \frac{۳\cos x}{(\sin x+۳)^۲}$ ۱٫۵ $\quad \cos(\frac{۵\pi}{۳}) = +\frac{۱}{۲}$

$(f\circ g)'(\ldots) = \left(-\frac{۱}{۵\sqrt{\frac{۱}{۲x^۵}+\frac{۱}{۲x^۵}}}\right)' = \left(-\frac{۱}{۵\sqrt{\frac{۱}{x^۵}}}\right)' = (-x)' = -۱$ ✓ ۲ ۵

$\frac{f-1}{x}=g$ $\lim_{x\to 0}\frac{f-1}{x}=\lim_{x\to 0} g$ $\frac{0}{0}$ hop $\lim_{x\to 0} f' = f'(0)$ $f(x)=\left(\frac{\sin x-1}{\sin x+1}\right)^2=\left(1-\frac{2}{\sin x+1}\right)^2$ $f'=2\left(1-\frac{2}{\sin x+1}\right)\left(\frac{-2\cos x}{(\sin x+1)^2}\right)$ $2\left(1-\frac{2}{1}\right)\left(\frac{-2(1)}{1}\right)=4$  ۱٫۵	۶
$y=-x^2-1$ $y'=-2x$  $-\sqrt{-1-d}$ $\sqrt{-d-1}$ d $+2\left(+\sqrt{-1-d}\right)(+2)\left(\sqrt{-1-d}\right)=+1$ $-d-1=\frac{5}{4}$ $d=-\frac{5}{4}$ ~ عرض $=\frac{5}{4}$ ۱	۷
$f(x)=2\sqrt{x}\,(4x^2+3)$ $f'(x)=\frac{2(4x^2+3)}{2\sqrt{x}}+\frac{2\times 8x\sqrt{x}\sqrt{x}}{\sqrt{x}}$ $=\frac{4x^2+3+16x^2}{\sqrt{x}}$ x: 0 ; f': $+$; f: 0 ↗ مماس قائم  شیب $+\infty$ ۱	۸
$f'(x)=\frac{\frac{-2x^2+x+1}{2\sqrt{x}}-\frac{(-4x+1)2x}{2\sqrt{x}}}{(-2x^2+x+1)^2}=\frac{6x^2-x+1}{2\sqrt{x}\,(\quad)^2}$ $-\frac{1}{2}$, 1 ; 0 x: 0, 1 ; f': $+$, $+$; f: 0 ↗ ↗  $\frac{0}{0}$ ۱	۹
$fog=\left(\frac{1}{\sqrt{x^2-1}}\left[\frac{1}{\sqrt{x^2+1}}\right]\right)^3$ $\lim_{x\to\frac{\sqrt{5}}{2}^-}\frac{\left(\frac{1}{\sqrt{x^2-1}}(2)\right)^3-f\left(\frac{\sqrt{5}}{2}\right)}{x-\frac{\sqrt{5}}{2}}\xrightarrow{hop}\lim_{x\to\frac{\sqrt{5}}{2}} 3\left(\frac{2}{\sqrt{x^2-1}}\right)^2\left(\frac{\cancel{2}\times 2x}{\cancel{2}\sqrt{x^2-1}}\right)=$ $\frac{1}{2}$ $\frac{1}{2}$ $-\frac{\cancel{3}\times(2\times 2)(2\times 2)\left(2\times 2\times\frac{\sqrt{5}}{\cancel{2}}\right)}{\cancel{3}\times 16\sqrt{5}}=-2$ ۱٫۵	۱۰

۳

$x=1 \longrightarrow y=\dfrac{۲+m}{۴}$

$y'=\dfrac{(\overset{m+۲}{\cancel{۲x+}m})(\overset{۴}{\cancel{x+۳}})-(\overset{m+۲}{x^{۲}\cancel{+mx}+1})}{\underset{۴^{۲}}{(\cancel{x+۳})}^{۲}}=\dfrac{۳m+۶}{۱۶}=\dfrac{۳}{۴} \rightsquigarrow \boxed{m=۲}$

$y=\dfrac{۳}{۴}x+\dfrac{n}{۴} \rightsquigarrow \dfrac{۳+n}{۴}=\dfrac{۲+۲}{۴} \rightsquigarrow \boxed{n=1}$

$\left.\right\} \boxed{m+n=۳}$

۴

$۳g-f(x)=\dfrac{۹}{۳+\sin x}-\dfrac{(۳-\cancel{\sin x})(۹+\sin^{۲}x+۳\sin x)}{(۳-\cancel{\sin x})(۳+\sin x)}=\dfrac{-\sin x(\cancel{\sin x}+۳)}{\cancel{\sin x}+۳}$

یا $-\sin x \xrightarrow{\text{مشتق}} (۳g-f)'(x)=-\cos x \rightsquigarrow -\cos\left(\dfrac{۴\pi}{۳}\right)=\boxed{-\dfrac{1}{۲}}$

۶

$g(x)=\dfrac{f(x)-1}{x} \rightarrow \lim\limits_{x\to 0} g(x)=f'(0)$

$f'(x)=\dfrac{۲}{(1+\sin x)^{۲}}\times \cos x\times ۲\left(\dfrac{\sin x-1}{1+\sin x}\right) \rightarrow f'(0)=\dfrac{۲}{1}\times 1\times -۲=\boxed{-۴}$

۸

$f(x)=۸x^{\frac{۵}{۲}}+۶x^{\frac{1}{۲}} \longrightarrow f'(x)=۲۰x^{\frac{۳}{۲}}+۳x^{-\frac{1}{۲}}$

معادله‌ی خط مماس در نقطه‌ی $x=a$ برابر است با:

$y-۲\sqrt{a}(۴a^{۲}+۳)=\dfrac{۲۰a^{۲}+۳}{\sqrt{a}}(x-a)$

$x,y=0 \rightsquigarrow \cancel{-}۲\sqrt{a}(۴a^{۲}+۳)=\dfrac{۲۰a^{۲}+۳}{\sqrt{a}}(\cancel{-}a) \rightsquigarrow ۲\cancel{a}(۴a^{۲}+۳)=۲۰a^{۲}+۳(\cancel{a})$

$۸a^{۲}+۶=۲۰a^{۲}+۳ \longrightarrow ۱۲a^{۲}=۳ \longrightarrow \boxed{a=\pm\dfrac{1}{۲}} \rightsquigarrow a>0 \rightsquigarrow a=\dfrac{1}{۲}$

$m=۲۰\left(۲^{-1\times\frac{۳}{۲}}\right)+۳\left(۲^{-1(-\frac{1}{۲})}\right)=\boxed{۸\sqrt{۲}}$

۹

$$y = mx \longrightarrow \frac{\sqrt{a}}{-2a^2+a+1} = ma \longrightarrow \frac{1}{-2a^2+a+1} = m\sqrt{a}$$

$$m\sqrt{a}\,(-2a^2+a+1) = 1 \longrightarrow -2m\left(a^{\frac{5}{2}}\right) + m\left(a^{\frac{3}{2}}\right) + m(a)^{\frac{1}{2}} = 1 \quad \xrightarrow{\text{مشتق}}$$

$$-5m\left(a^{\frac{3}{2}}\right) + \frac{3}{2}m\left(a^{\frac{1}{2}}\right) + \frac{m}{2}\left(a^{-\frac{1}{2}}\right) = 0$$

$$\frac{m}{2}\left(a^{-\frac{1}{2}}\right)\left(-10a^2+3a+1\right) = 0 \longrightarrow a = -\frac{1}{5} \ \text{یا}\ a = \frac{1}{2} \ \checkmark \quad (a>0)$$

$$f(a) = \frac{\frac{\sqrt{2}}{2}}{-2\left(\frac{1}{4}\right)+\frac{1}{2}+1} = \frac{\frac{\sqrt{2}}{2}}{1} = \boxed{\frac{\sqrt{2}}{2}}$$

۱۰

$$g(x) = (x^2-1)^{-\frac{1}{2}} \longrightarrow g'(x) = -\frac{1}{2}(2x)(x^2-1)^{-\frac{3}{2}}$$

$$g'\left(\frac{\sqrt{5}}{2}\right) = -\frac{1}{2}(\sqrt{5})\left(\frac{5}{4}-1\right)^{-\frac{3}{2}} \longrightarrow -\frac{\sqrt{5}}{2}\left(\underset{8}{\cancel{2^{-2\left(-\frac{3}{2}\right)}}}\right) = -4\sqrt{5}$$

$$g\left(\frac{\sqrt{5}}{2}^-\right) = \frac{1}{\sqrt{\frac{5}{4}^- - 1}} = \frac{1}{\sqrt{\frac{1}{4}^-}} = \frac{1}{\frac{1}{2}^-} = 2^+$$

$$f'(2^+) = \left((2x)^3\right)' = 24x^2 = 24\times 4$$

$$f\circ g'\left(\frac{\sqrt{5}}{2}^-\right) = -4\sqrt{5}\times 24\times 4 \quad \xrightarrow{\div -48\sqrt{5}} \quad \frac{\overset{2}{\cancel{24}}\times\cancel{4}\times -4\sqrt{5}}{-\cancel{4}8\sqrt{5}} = \boxed{8}$$