نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ۲۴ ..... کلاس دوازدهم ریاضی .....

A۱۷۵ آزمون

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$F(3)=5$

$d: (3,5),(0,1) \rightarrow m=\frac{1-5}{0-3}=\frac{4}{3}$ $\quad F'(3)=m=\frac{4}{3}$

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$A(b,\frac{1}{2}b+\frac{5}{2})\ d \rightarrow (3,3),(-1,1)$ $\quad y-3=\frac{1}{2}(x-3) \rightarrow y=\frac{1}{2}x+\frac{5}{2}$

$F'(A)=m_d$ , $F(A)=\frac{1}{2}b+\frac{5}{2}$

$\frac{a}{2\sqrt{ab-1}}=\frac{1}{2} \rightarrow 2a=2\sqrt{ab-1} \rightarrow 4a^2=4(ab-1) \rightarrow b=\frac{4a-4}{a}$

$b=\frac{4a-4}{a} \rightarrow 2a=2\sqrt{\frac{4a^2-4a-2}{2}}$ توان ۲ $\rightarrow 4a^2=16a^2-16a-4 \rightarrow 4a^2-16a-4=0$ $\begin{cases} a=4 \\ a=-\frac{1}{4} \text{ قبول نیست} \end{cases}$

$F(x)=\sqrt{2x-1} \rightarrow F(4)=\sqrt{7}=3$

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$y=\frac{3}{4}x+\frac{5}{4} \rightarrow m=\frac{3}{4}$

$\frac{x^2+mx+1}{x+3} \xrightarrow{\text{مشتق}} \frac{(2x+m)(x+3)-(1)(x^2+mx+1)}{(x+3)^2}=\frac{(2+m)(4)-(1)(3+m)}{16}$

$\frac{8+4m-3-m}{16}=\frac{3}{4} \rightarrow \frac{5+3m}{4}=3 \rightarrow 3+m=4 \rightarrow m=1$

$y=\frac{x^2+2x+1}{x+3}$ $\xrightarrow{x=1}$ $1$ $\quad (1,1)$ در خط صدق می‌کند $\rightarrow 4(1)-3(1)=n \rightarrow n=1$

$m+n=3$

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$F(x)=\frac{(3-\sin\alpha)(9+3\sin\alpha+\sin^2\alpha)}{(3-\sin\alpha)(3+\sin\alpha)}=\frac{9+3\sin\alpha+\sin^2\alpha}{3+\sin\alpha}$

$g(x)=\frac{3}{3+\sin\alpha}$

$(3g-F)(x)=\frac{-3\sin\alpha-\sin^2\alpha}{3+\sin\alpha}$

$\frac{-\sin\alpha(\sin\alpha+3)}{(3+\sin\alpha)}=-\sin\alpha \xrightarrow{\text{مشتق می‌گیریم}} -\cos\alpha=-\cos\frac{5\pi}{3}=\frac{1}{2}$

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$(fog)'(x)=g'(x)\cdot f'(g(x))$ $\xrightarrow{x=\sqrt{3}}$

$f(g(\sqrt{3}))=-\frac{1}{\sqrt{\frac{1}{x^5+|x^5|}+\left|\frac{1}{x^5+|x^5|}\right|}}=\frac{-1}{\sqrt{\frac{x}{2x^5}}}=\frac{-1}{\sqrt{\frac{1}{x^4}}}=\frac{-1}{\frac{1}{x^2}}=-x^2$

$\xrightarrow{\text{مشتق}}$ $-1$

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$F(x) = x.g(x) + 1 \xrightarrow{F(0)} g(x) = \frac{F(x)-1}{x}$

$\lim_{x\to 0} g(x) = \frac{F(x)-1}{x-0} = F'(0)$

$F'(x) = 2\left(\frac{-1+\sin x}{1+\sin x}\right).\frac{(\cos x)(1+\sin x)-(\cos x)(\sin x-1)}{(1+\sin x)^2} \xrightarrow{x=0} 2\left(\frac{-1}{1}\right).\frac{(1)(1)-(1)(-1)}{(1+0)^2}$

$= (-2).(2) = -4$ ✓ ج

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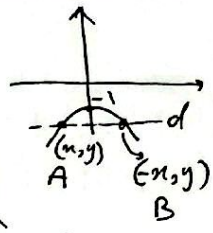
$y = x^2+1 \xrightarrow[\text{نسبت به محور } x]{\text{قرینه}} y = -x^2-1$

$m_d = 0 \to y = b < -1$

مشتق $\to -2x$

$(-2x_A)(2x_B) = -1 \to -4x^2 = 1 \to x^2 = \frac{1}{4} \to x = \frac{1}{2}$

وقتی نقط A را بنامیم / وقتی نقط B را بنامیم




$\frac{1}{2}$ فاصله از مبدا مختصات $= \left|-\frac{1}{4}\right| = \frac{1}{4}$

طول معادله $\xrightarrow{\text{جای}}$ $y = -x^2-1 \to y = -\frac{1}{4}-1 = -\frac{5}{4}$ $\xrightarrow{\text{نقطه}}$ $y = -\frac{5}{4}$ ✓ ب

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$d$ بیضی $\to y = ax$

$F(x) = 2\sqrt{x}(4x^2+3)$

$F'(x) = \left(\frac{1}{2\sqrt{x}}\right)(4x^2+3) + (8x)(2\sqrt{x})$

$F(b) = ab \xrightarrow{\left(\frac{1}{2\sqrt{b}}\right)} (4b^2+3) + (8b)(2\sqrt{b}) = a = \frac{2(4b^2+3)}{\sqrt{b}}$ *

$F'(b) = a \to (4b^2+3) = \frac{ab}{2\sqrt{b}} = \frac{a\sqrt{b}}{2} \to a = \frac{2(4b^2+3)}{\sqrt{b}}$

* طرفین و وسطین $\to (1)(4b^2+3)+16b^2 = 2(4b^2+3) \to 16b^2 = 4b^2+6 \to 12b^2 = 6 \to b = \frac{1}{2}$ ✓ د

شرط طاعه رد بیضی

$d$ بیضی $= a = \frac{2(4b^2+3)}{\sqrt{b}} \to a = \frac{2(1+3)}{\frac{1}{\sqrt{2}}} = 8\sqrt{2}$

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$d$ بیضی $y = ax \xrightarrow{A \text{ نقطه}} (m, am)$

$F(m) = am \to \frac{\sqrt{m}}{-2m^2+m+1} = a\sqrt{m} \to a = \frac{1}{\sqrt{m}(-2m^2+m+1)}$

$\frac{1}{\sqrt{\frac{1}{2}\left(-2\left(\frac{1}{4}\right)+\frac{1}{2}+1\right)}} = \sqrt{2}$ ✓ ج

$F'(m) = a$

$F'(m) = \frac{\left(\frac{1}{2\sqrt{m}}\right)(-2m^2+m+1)-(-4m+1)(\sqrt{m})}{(-2m^2+m+1)^2} = \frac{1}{\sqrt{m}(-2m^2+m+1)}$

$\to \left(-m^2+\frac{m}{2}+\frac{1}{2}\right)+4m^2-m = -2m^2+m+1 \to 5m^2-\frac{3}{2}m-\frac{1}{2} = 0 \to 10m^2-3m-1 = 0$

$m = \frac{1}{2}$ ✓ ج

$m = -\frac{1}{5}$ ✗ ق ق

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$g'\left(\frac{\sqrt{5}}{2}\right) \times F'\left(g\left(\frac{\sqrt{5}}{2}\right)\right)$

$F(x) = (x[x])^3 \xrightarrow{x=2^+} (2x)^3 = 8x^3$ مشتق $\to 4x^2 = F'(x)$

$g\left(\frac{\sqrt{5}}{2}^-\right) = \frac{1}{\sqrt{\left(\frac{2}{3}\right)^{-1}-1}} = \frac{1}{\sqrt{\frac{1}{4}}} = 2^+$

$F'(2^+) = 96$

$g(x) = \frac{1}{\sqrt{x^2-1}} \xrightarrow{\text{مشتق}} g'(x) = \frac{-1}{x^2-1} \times \frac{2x}{2\sqrt{x^2-1}} = \frac{-x}{(x^2-1)(\sqrt{x^2-1})}$ ✓ ب

$g'\left(\frac{\sqrt{5}}{2}^-\right) \times F'(2^+) = (-4\sqrt{5})(96) \to \frac{-384\sqrt{5}}{64+16\sqrt{5}} = -7$

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$$y = mn \longrightarrow \frac{\sqrt{a}}{-2a^2+a+1} = ma \longrightarrow \frac{1}{-2a^2+a+1} = m\sqrt{a}$$

$$m\sqrt{a}(-2a^2+a+1) = 1 \longrightarrow -2m(a^{\frac{5}{2}}) + m(a^{\frac{3}{2}}) + m(a)^{\frac{1}{2}} = 1$$ مشتق

$$-5m(a^{\frac{3}{2}}) + \frac{3}{2}m(a^{\frac{1}{2}}) + \frac{m}{2}(a^{-\frac{1}{2}}) = 0$$

$$\frac{m}{2}(a^{-\frac{1}{2}})(-10a^2+3a+1) = 0 \longrightarrow a = -\frac{1}{5} \text{ ی } a = \frac{1}{2} \quad (a>0)$$

$$\phi'(a) = \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$$