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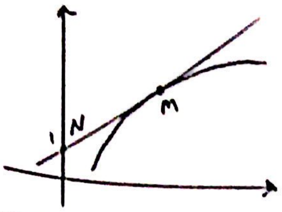

$N\begin{vmatrix}0\\1\end{vmatrix} \quad M\begin{vmatrix}3\\5\end{vmatrix}$

$m = \dfrac{5-1}{3-0} = \dfrac{4}{3} \longrightarrow f'(3) = m = \dfrac{4}{3}$

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$f(x) = \sqrt{ax-1} \qquad B\begin{vmatrix}-1\\1\end{vmatrix} \quad C\begin{vmatrix}2\\2\end{vmatrix} \qquad m = \dfrac{2-1}{2-(-1)} = \dfrac{1}{3}$

معادله خط: $y-1 = \dfrac{1}{3}(x+1)$

معادله خط: $y = \dfrac{1}{3}x + \dfrac{4}{3}$

$\dfrac{1}{3}x + \dfrac{4}{3} = \sqrt{ax-1} \longrightarrow x+4 = 3\sqrt{ax-1}$

$x^2 + (8-9a)x + 25 = 0$

تماس = ریشه مضاعف، $\Delta = 0$

$(x \pm 5)^2 = 0 \quad \begin{cases} 8-9a = 10 & a = \dfrac{-2}{9} \; \times \\ 8-9a = -10 & a = 2 \; \checkmark \end{cases}$

$f(x) = \sqrt{2x-1} \xrightarrow{x=5} \sqrt{(2)(5)-1} = 3$

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$y = \dfrac{x^2+mx+1}{x+3} \qquad y' = \dfrac{3(m+2)}{16} \qquad \dfrac{3(m+2)}{16} = \dfrac{3}{4} \qquad m+2 = 4 \quad m=2$

$4y = 3x+n \longrightarrow m = \dfrac{3}{4} \qquad y = \dfrac{x^2+2x+1}{x+3} \xrightarrow{x=1} \begin{vmatrix}1\\1\end{vmatrix} \qquad m+n = 2+1 = 3$

$4 = 3+n \qquad n=1$

۴

$f(x) = \dfrac{27 - \sin^3 x}{9 - \sin^2 x} = \dfrac{\cancel{(3-\sin x)}(9 + 3\sin x + \sin^2 x)}{\cancel{(3-\sin x)}(3+\sin x)} = \dfrac{9+3\sin x + \sin^2 x}{3+\sin x}$

$(3g-f)' = \left(\dfrac{9}{\sin x+3} - \dfrac{9+3\sin x+\sin^2 x}{3+\sin x}\right)' = \left(-\left(\dfrac{\sin^2 x + 3\sin x}{3+\sin x}\right)\right)' = (-\sin x)'$

$(3g-f)' = -\cos x \xrightarrow{x = \frac{5\pi}{3}} -\cos\dfrac{5\pi}{3} = \dfrac{-1}{2}$

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$g'(\sqrt[5]{3})\, f'(g(\sqrt[5]{3})) = (f\circ g(x))' \qquad x>0 \qquad g(x) = \dfrac{1}{2x^5} \qquad f(x) = \dfrac{-1}{\sqrt[5]{2x}}$

$f\circ g(x) = f(g(x)) = -x \qquad (f\circ g)'(x) = -1$

۶

$f(x) = \left(\dfrac{\sin x - 1}{\sin x + 1}\right)^2 = \left(1 + \dfrac{-2}{\sin x+1}\right)^2 = 1 + \dfrac{4}{(1+\sin x)^2} + \dfrac{-4}{\sin x+1} = 1 + xg(x)$

$xg(x) = \dfrac{4}{(1+\sin x)^2} - \dfrac{4}{\sin x+1} = \dfrac{-4}{\sin x+1}\left(\dfrac{\sin x}{\sin x+1}\right)$

$\lim\limits_{x\to 0} g(x) = \dfrac{\dfrac{-4}{\sin x+1}\left(\dfrac{\sin x}{\sin x+1}\right)}{x} \xrightarrow{\sin x \sim x} \dfrac{-4}{(\sin x+1)^2} = -4$

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$y=x^2+1 \xrightarrow{\text{قرینه نسبت به محور } x} y=-x^2-1 \longrightarrow y'=-2x$

$A\left|\begin{matrix}\alpha\\ \beta\end{matrix}\right. \quad B\left|\begin{matrix}-\alpha\\ \beta\end{matrix}\right. \qquad m_1=-2\alpha \quad m_2=+2\alpha \qquad m_1m_2=-1$ عمود $\qquad -4\alpha^2=-1 \quad \alpha=\pm\frac{1}{2}$

$A\left|\begin{matrix}\frac{1}{2}\\ \beta\end{matrix}\right. \quad B\left|\begin{matrix}\frac{-1}{2}\\ \beta\end{matrix}\right. \qquad y=-\left(\pm\frac{1}{2}\right)^2-1=\frac{-5}{4}$

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$f(x)=2\sqrt{x}\,(4x^2+3) \qquad f'(x)=\frac{1}{\sqrt{x}}(4x^2+3)+(8x)(2\sqrt{x})=\frac{20x^2+3}{\sqrt{x}}$

$y-y_0=m(x-x_0) \longrightarrow y-(2\sqrt{\alpha}(4\alpha^2+3))=\frac{20\alpha^2+3}{\sqrt{\alpha}}(x-\alpha) \xrightarrow{(0,0)}$

$\alpha^2=\frac{1}{4} \qquad \alpha \to \frac{1}{2}\ ✓,\ \frac{-1}{2}\ ✗ \qquad m=\frac{20\alpha^2+3}{\sqrt{\alpha}}=\frac{20\left(\frac{1}{4}\right)+3}{\sqrt{\frac{1}{2}}}=8\sqrt{2}$

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$f(x)=\frac{\sqrt{x}}{-2x^2+x+1} \Rightarrow m\alpha=\frac{\sqrt{\alpha}}{-2\alpha^2+\alpha+1} \qquad -2m\alpha^{\frac{5}{2}}+m\alpha^{\frac{1}{2}}+m\alpha^{\frac{3}{2}}=1$

$y=mx$

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$(fog)'(x)=g'\left(\frac{\sqrt{5}}{2}\right)f'\left(g\left(\frac{\sqrt{5}}{2}\right)\right) \qquad g(x)=\frac{1}{\sqrt{x^2-1}} \qquad f(x)=(x[x])^3$

$g(x)=(x^2-1)^{\frac{-1}{2}} \Rightarrow g'(x)=\frac{-1}{2}(x^2-1)^{\frac{-3}{2}}\times 2x \longrightarrow g\left(\frac{\sqrt{5}^-}{2}\right)=2^+$

$f'(2^+)= \qquad ((2x)^3)'=(8x^3)' \longrightarrow 24x^2=24\times 4=96$

$(fog)'(x)=g'\left(\frac{\sqrt{5}^-}{2}\right)\times f'\left(g\left(\frac{\sqrt{5}^-}{2}\right)\right)=-4\sqrt{5}\times 4\times 24 \qquad \frac{24\times 4\times(-4\sqrt{5})}{(-48\sqrt{5})}=8$

[illegible] $-48\sqrt{5}$