تکلیف ۳۴ [illegible]

---

(۱)

$y=ax+b \Rightarrow f'(3)=a$ $\quad (0,1)\ (3,5)$ $\quad \frac{5-1}{3-0}=\frac{4}{3}$

$y=\frac{4}{3}x+b \Rightarrow f'(3)=\frac{4}{3}$

---

(۲)

$(-1,1)\ (2,2)$ $\quad \frac{2-1}{2-(-1)}=\frac{1}{3}$ $\quad f'(8)=\frac{1}{3}$ $\quad f'(x)=\frac{a}{2\sqrt{ax-1}}$

$y=ax+b \Rightarrow y=\frac{1}{3}x+\frac{4}{3}$

$f(8)=\frac{8}{3}+\frac{4}{3}=\sqrt{8a-1}$ $\Big\} \Rightarrow f'(8)=\frac{a}{2\sqrt{8a-1}}=\frac{a}{2\left(\frac{8}{3}+\frac{4}{3}\right)}=\frac{1}{3}$

---

(۳)

$f(x)=\frac{x^2+mx+1}{x+3}$ $\quad f'(x)=\frac{x^2+6x+3m-1}{(x+3)^2} \Rightarrow f'(1)=\frac{7+3m-1}{16}=\frac{3}{4}$

$\begin{matrix} 1 & m & 1 \\ 0 & 1 & 3 \end{matrix}$ $\quad 4y-3x=n \Rightarrow 4y=3x+n \Rightarrow y=\frac{3}{4}x+n$ $\quad f'(1)=\frac{3}{4}$

$\Rightarrow 12=7+3m-1 \Rightarrow 3m=6 \quad m=2]$ $\quad f(1)=\frac{1+2+1}{4}=1]$

$f(1)=1$ $\quad y=\frac{3}{4}x+\frac{n}{4} \Rightarrow y=\frac{3}{4}\times 1+\frac{n}{4}=1 \Rightarrow n=1$ $\quad m+n=2+1=3]$

---

(۴)

$3g'\left(\frac{5\pi}{3}\right)-f'\left(\frac{5\pi}{3}\right)=(3g-f)'$ $\quad \frac{3\times 3}{3+\sin x}-\frac{27-\sin^3 x}{9-\sin^2 x}=$

$\frac{9}{3+\sin x}-\frac{\cancel{(3-\sin x)}(9+\sin^2 x+3\sin x)}{\cancel{(3-\sin x)}(3+\sin x)}=\frac{9-9-\sin^2 x-3\sin x}{3+\sin x}=-\frac{\sin x(\sin x+3)}{3+\sin x}, -\sin x$

$(3g-f)'=(-\sin x)'=-\cos x \Rightarrow (3g-f)'\left(\frac{5\pi}{3}\right)=-\cos\frac{5\pi}{3}=\frac{1}{2}]$

---

(۵)

$g'(\sqrt[5]{3})\,f'(g(\sqrt[5]{3}))=(fog)'(x) \Rightarrow fog(x)=\frac{-1}{\sqrt[5]{\frac{1}{x^5+|x^5|}+\left|\frac{1}{x^5+|x^5|}\right|}}$

$\Rightarrow fog(x)=\frac{-1}{\frac{\sqrt[5]{2}}{\sqrt[5]{2x^5}}}=\frac{-1}{\frac{\sqrt[5]{2}}{x\sqrt[5]{2}}} \Rightarrow fog(x)=-x^5 \Rightarrow (fog)'(\sqrt[5]{3})=$

$(fog)'(x)=-5x^4 \Rightarrow fog'(\sqrt[5]{3})=-5\sqrt[5]{81}$

---

(۶)

$f(x)=x\,g(x)+1 \Rightarrow \frac{f(x)-1}{x}=g(x) \Rightarrow g(x)=\frac{\frac{(\sin x-1)^2}{(\sin x+1)^2}-1}{x} \Rightarrow$

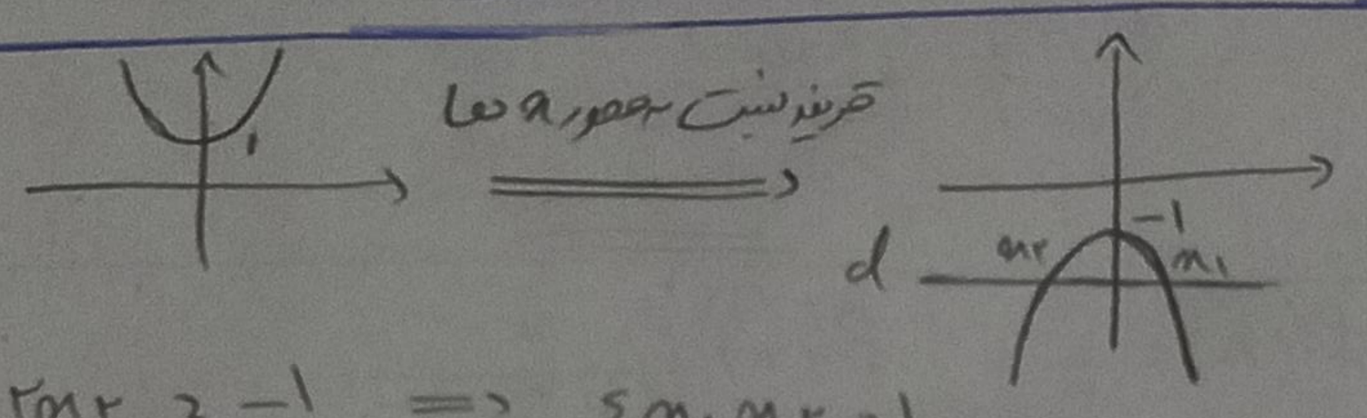
$g(x)=\frac{\sin^2 x+1-2\sin x-\sin^2 x-1-2\sin x}{x(\sin x+1)^2} \Rightarrow g(x)=\frac{-4\sin x}{x(\sin x+1)^2}$

$\lim_{x\to 0} g(x)=\frac{-4\sin x}{x}\times\frac{1}{(\sin x+1)^2}=-4\times\frac{1}{1}=-4]$

سوگند زارعیان — کلیت ۳۴

۱۲

$y=x^2+1$ قرینه نسبت به محور x ها $\Longrightarrow$ $y=-x^2-1$, $y'=-2x$



فاصله‌ی خط d از محور xها برابر $\frac{5}{4}$ است

$-2x_1 \times -2x_2 = -1 \Rightarrow 4x_1x_2=1$

$\left.x_1x_2=-\frac{1}{4}\right]$ $x_1+x_2=0 \Rightarrow x_1=-x_2 \Rightarrow x_1=\frac{1}{2}\quad x_2=-\frac{1}{2}$

$f(\frac{1}{2})=f(-\frac{1}{2})=-(\frac{1}{2})^2-1 \Rightarrow \left. f(\frac{1}{2})=f(-\frac{1}{2})=-\frac{5}{4}\right]$

۱۳

$d: 8x$ $f'(x)=\frac{2}{2\sqrt{x}}(4x^2+3)+2\sqrt{x}\,(8x)$

$\frac{f(a)-0}{a-0}=f'(a) \Rightarrow a f'(a)=f(a) \Rightarrow \frac{a}{\sqrt{a}}(4a^2+3)+2\sqrt{a}\,(8a)=2\sqrt{a}\,(4a^2+3)$

$\sqrt{a}\,(4a^2+3+16a)=2\sqrt{a}\,(4a^2+3) \Rightarrow a=\frac{1}{2} \Rightarrow f'(a)=8=f'(\frac{1}{2})$

$f'(\frac{1}{2})=\sqrt{2}\,(4)+\frac{2}{\sqrt{2}}(4)=8\sqrt{2}$ شیب خط d برابر $8\sqrt{2}$ است.

۱۴

$d=ax$ $f'(x)=\dfrac{\frac{1}{2\sqrt{x}}(-2x^2+x+1)-\sqrt{x}\,(-4x+1)}{(-2x^2+x+1)^2}=a$ *

$A:(x,y)$ $ax=\dfrac{\sqrt{x}}{-2x^2+x+1} \Rightarrow a\sqrt{x}\,(-2x^2+x+1)=1$

* $\dfrac{\frac{1}{2\sqrt{x}}(-2x^2+x+1)-\sqrt{x}\,(-4x+1)}{(-2x^2+x+1)^2}\times\sqrt{x}\,(-2x^2+x+1)=1 \Rightarrow \dfrac{\frac{1}{2}(-2x^2+x+1)+4x^2-x}{-2x^2+x+1}=1$

$\Rightarrow -x^2+\frac{1}{2}x+\frac{1}{2}+4x^2-x=-2x^2+x+1 \Rightarrow 5x^2-\frac{3}{2}x-\frac{1}{2}$ $x=\frac{5}{10}$, $x=-\frac{2}{10}$ در دامنه نیست

$x=\frac{5}{10}=\frac{1}{2}$ $\left. f(\frac{1}{2})=\dfrac{1}{\sqrt{2}(-\frac{1}{2}+\frac{1}{2}+1)}=\frac{\sqrt{2}}{2}\right]$

۱۵

$g'(x)=-\frac{1}{2}(2x)(x^2-1)^{-\frac{3}{2}} \Rightarrow g'(x)=-x(x^2-1)^{-\frac{3}{2}}$

$g'_-(\frac{\sqrt{5}}{2})=-\frac{\sqrt{5}}{2}(\frac{5}{4}-1)^{-\frac{3}{2}}=\left.-\frac{\sqrt{5}}{2}\times 8=-4\sqrt{5}\right]$

$g(\frac{\sqrt{5}}{2})=\dfrac{1}{\sqrt{x^2-1}}=\dfrac{1}{\sqrt{\frac{5}{4}-1}}=\dfrac{1}{(\frac{1}{2})}=2^+$ $x\to 2^+\ [x]=2$

$f(x)=(2x)^3=8x^3$

$f'(x)=24x^2$

$(fog)'=g'(x)\,f'(g(x))$

$(fog)'_-(\frac{\sqrt{5}}{2})=-4\sqrt{5}\times 24\times 4=\left.-16\times 24\sqrt{5}\right]$

$\left.\dfrac{-16\times 24\sqrt{5}}{48}=-8\sqrt{5}\right]$