نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ..... کلاس .....

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شیب خط مماس $f(x)$ را به ما می‌دهد:

$f(0)=1 , f(3)=5$

شیب خط مماس $= \frac{y_2-y_1}{x_2-x_1} = \boxed{\frac{4}{3}}$

۲

شیب خط مماس بر منحنی $= \frac{2-1}{2+1} = \frac{1}{3} \Rightarrow$ معادله خط $= y = \boxed{\frac{1}{3}x + \frac{4}{3}}$

$\frac{1}{3}x + \frac{4}{3} = \sqrt{ax-1} \xrightarrow{\text{یک نقطه برخورد}} x^2 + (8-9a)x + 25 = 0 \xrightarrow{\Delta=0}$

$(8-9a)^2 - 100 = 0 \Rightarrow a = 2 ; \frac{-2}{9} \Rightarrow f(x) = \sqrt{2x-1} \Rightarrow f(a) = \boxed{3}$

۳

$y = \frac{3}{4}x + \frac{5}{4}$ و $f'(1) = \frac{3}{4}$

$\frac{(2x+m)(x+3) - 1(x^2+mx+1)}{(x+3)^2} \Rightarrow \frac{(2(1)+m)(1+3) - (1^2+m(1)+1)}{(1+3)^2} = \frac{3}{4} \Rightarrow \frac{2+m}{16} = \frac{1}{4}$

$\Rightarrow m+2=4 \Rightarrow m=2 \Rightarrow y(1) = \frac{4}{4} = 1 \xrightarrow[\text{نقطه}]{\text{تابع}} (1,1) \Rightarrow f(1) - 3(1) = n \Rightarrow n=1 \Rightarrow m+n = \boxed{3}$

۴

$f(x) = \frac{(3-\sin x)(9+3\sin x+\sin^2 x)}{(3-\sin x)(3+\sin x)} = \frac{9+3\sin x+\sin^2 x}{3+\sin x}$

$\underbrace{3g(x)-f(x)}_{h} = 3\left(\frac{3}{3+\sin x}\right) - \frac{9+3\sin x+\sin^2 x}{3+\sin x} = \frac{-3\sin x - \sin^2 x}{3+\sin x} = -\sin x$

$h' = -\cos x = -\cos\left(2\pi - \frac{\pi}{3}\right) = \boxed{-\frac{1}{2}}$

۵

سوال از $(fog)'\left(\sqrt[3]{3}\right)$

$|x|, |x^3| = x, x^3$ (هم علامت) $\Rightarrow f(x) = \frac{-1}{\sqrt[3]{2x}} , g(x) = \frac{1}{2x^3}$

$f(g(x)) = \frac{-1}{\sqrt[3]{2 \times \frac{1}{2x^3}}} \Rightarrow f(g(x)) = -x \xrightarrow{\text{مشتق}} (f(g(x))' = \boxed{-1}$

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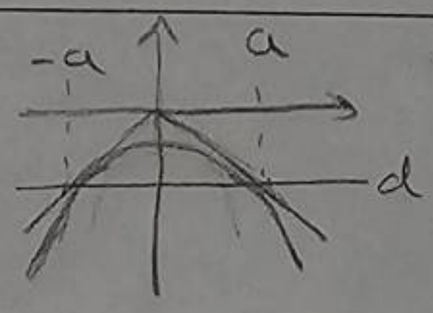
$x(g(x)) = f(x) - 1 \Rightarrow g(x) = \frac{f(x)-1}{x} \Rightarrow g(x) = \frac{\left(\frac{-1+\sin x}{1+\sin x}\right)^2 - 1}{x}$

$f(x) - 1 = \left(\frac{-1+\sin x}{1+\sin x} - 1\right)\left(\frac{-1+\sin x}{1+\sin x} + 1\right) \xrightarrow[\text{مزدوج}]{\text{ضرب در}} \left(\frac{-2}{1+\sin x}\right)\left(\frac{2\sin x}{1+\sin x}\right) = \frac{-4\sin x}{(1+\sin x)^2}$

$\lim_{x\to 0} g(x) = \frac{\frac{-4\sin x}{(1+\sin x)^2}}{x} = \lim_{x\to 0}\left(\frac{-4}{(1+\sin x)^2} \cdot \frac{\sin x}{x}\right) = -4 \Rightarrow \lim_{x\to 0} g(x) = -4 \times 1 = \boxed{-4}$ ۶

$y = x^2 + 1 \xrightarrow[x]{\text{قرینه نسبت به}} -x^2 - 1 \Rightarrow f'(x) = -2x$



$f'(a)f'(-a) = -1 \Rightarrow (-2a)(2a) = -1 \overset{a>0}{\Rightarrow} a = \frac{1}{2}$

$f\left(\frac{1}{2}\right) = -\left(\left(\frac{1}{2}\right)^2 + 1\right) = \frac{-5}{4} \rightarrow y = \frac{-5}{4}$ خط d تابع ثابت است $\Rightarrow$

$\Downarrow$

فاصله از مبدا مختصات $\frac{5}{4}$ است. ۲

$f'(x) = \frac{\frac{1}{2\sqrt{x}}(-2x^2+x+1) - (-4x+1)\sqrt{x}}{(-2x^2+x+1)^2} \Rightarrow \frac{\sqrt{x}}{x} = \frac{\frac{-2x^2+x+1}{2\sqrt{x}} - (-4x+1)\sqrt{x}}{(-2x^2+x+1)^2} \Rightarrow \frac{\sqrt{x}}{x} = \frac{-2x^2+x+1-8x(-4x+1)}{2\sqrt{x}(-2x^2+x+1)}$

$\Rightarrow 2 = \frac{32x^2 - x + 1}{-2x^2+x+1} \Rightarrow 10x^2 - 3x - 1 = 0$

$x = \frac{-1}{5}$ ق ق ق $\frac{1}{2}$ ق ق ق

$f\left(\frac{1}{2}\right) = \frac{\frac{\sqrt{2}}{2}}{-2\left(\frac{1}{2}\right)+\frac{1}{2}+1} = \boxed{\frac{\sqrt{2}}{2}}$

مبدأ آنالز است $\Rightarrow \frac{f(x)}{x} = f(x)$ ۱

$f(x) = g(x) \rightarrow$ در x_0 $\Rightarrow 2\sqrt{x}(4x^2+3) = mx \Rightarrow 8x^2 + 4 = m\sqrt{x}$

$f'(x) = g'(x) \rightarrow 16x = \frac{m}{2\sqrt{x}} \Rightarrow 8x^2 + 4 = 32x^2 \overset{x>0}{\Rightarrow} x = \frac{1}{2}$ $\xrightarrow[x=\frac{1}{2}]{\text{جایگذاری}}$ $m = 32 \times \sqrt{\frac{1}{2}}$

$\boxed{m = 8\sqrt{2}}$ ۹

$x \to \left(\frac{\sqrt{5}}{2}\right)^- \Rightarrow \sqrt{x^2-1} \rightarrow \left(\frac{1}{2}\right)^- \Rightarrow g(x) \rightarrow 2^+$

$(f(g(x)))' = g'\left(\frac{\sqrt{5}}{2}\right) \cdot f'_+(2)$

$g'(x) = \left((x^2-1)^{-\frac{1}{2}}\right)' = -\frac{1}{2}(x^2-1)^{-\frac{3}{2}}(2x) \Rightarrow g'\left(\frac{\sqrt{5}}{2}\right) = -4\sqrt{5}$ $\xleftarrow{x>0}$

$x \to 2^+ \Rightarrow f(x) = (8x)^2 \Rightarrow f'(x) = 128x \Rightarrow f'_+(2) = 96$

$(-4\sqrt{5})(96) = 8(-48\sqrt{5})$ ۱۰

$\boxed{\text{درست}}$

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