Subject:
Year. Month. Date. ( )

۱۴ آذر

عارفه پورصبری - تکلیف ۲۴

① $f'(3) =$ شیب خط مماس $\begin{vmatrix}3\\5\end{vmatrix}$ $\begin{vmatrix}0\\1\end{vmatrix}$ $\Rightarrow m = \frac{\Delta y}{\Delta x} = \frac{5-1}{3-0} = \boxed{\frac{4}{3}}$ جواب ۲

② $m = \frac{1}{3}$ $f'(x) = \frac{a}{2\sqrt{ax-1}} = \frac{1}{3} \Rightarrow 3a = 2\sqrt{ax-1} \rightarrow 9a^2 = 4ax-4$

$y = \frac{1}{3}x + \frac{4}{3}$ $\sqrt{ax-1} = \frac{1}{3}x + \frac{4}{3} \Rightarrow ax-1 = \frac{1}{9}(x^2+8x+16)$

$0 = \frac{1}{9}x^2 + (\frac{8}{9}-a)x + \frac{16}{9}+1 \rightarrow \Delta = 0 \rightsquigarrow (\frac{8}{9}-a)^2 - \frac{100}{81} = a^2 + \frac{64}{81} - \frac{16}{9}a - \frac{100}{81} = 0$

$0 = a^2 - \frac{16}{9} - \frac{36}{81} \Rightarrow 9a^2 - 16a - 4 = 0 \rightsquigarrow (a-2)(a+\frac{2}{9})$ $a \to 2$, $a \to -\frac{2}{9}$ غ ق ق $\Rightarrow \boxed{f(a) = 3}$ ۲

③ $m = \frac{3}{4} = f'(1) \Rightarrow y' = \frac{(2x+m)(x+c)-(x^2+mx+1)}{(x+c)^2} \rightarrow f'(1) = \frac{4(2+m)-(2+m)}{16}$ جواب ۲

$\frac{8+4m-2-m}{16} = \frac{3m+6}{16} = \frac{3}{4} \Rightarrow \boxed{m=2}$ $f(1) = 1 = \frac{3+n}{4} \rightarrow \boxed{n=1}$ $n+m = \boxed{3}$

④ $f(x) = \frac{9+\sin^2 x+3\sin x}{3+\sin x} \Rightarrow 3g(x) - f(x) = \frac{-(\sin^2 x+3\sin x)}{3+\sin x} = -\sin x$

مشتق $\rightarrow -\cos x \Rightarrow -\cos(\frac{5\pi}{3}) = \boxed{-\frac{1}{2}}$ جواب ۲

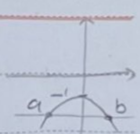
⑤ $fog(x) \Rightarrow \frac{1}{\sqrt[5]{\frac{2}{2x^5}}} = -x$ مشتق $\boxed{-1}$

$f(x) = \frac{-1}{\sqrt[5]{2x}}$, $g(x) = \frac{1}{2x^5}$ ۲

⑥ $f(x) = \frac{\sin^2 x - 2\sin x + 1}{\sin^2 x + 2\sin x + 1} = x\,g(x) + 1 \rightsquigarrow \frac{\sin^2 x - 2\sin x + 1 - \sin^2 x - 2\sin x - 1}{x(\sin^2 x + 2\sin x + 1)} = g(x)$

$g(x) = \frac{-4\sin x}{x(\sin x+1)^2}$ $\lim_{x\to 0} g(x) \xrightarrow{\text{هم‌ارزی}} \frac{-4x}{x(x+1)^2} = \boxed{-4}$ جواب ۲

⑦ $f(x) = -x^2 - 1 \rightsquigarrow f'(x) = -2x \xrightarrow{\text{عمود}} \frac{1}{2x}$



$\begin{vmatrix}a\\-a^2-1\end{vmatrix}$ $\begin{vmatrix}b\\-b^2-1\end{vmatrix}$ $-2a = \frac{-1}{2b} \Rightarrow -4ab = 1$ $-a^2-1 = -b^2-1 \Rightarrow b^2 = a^2$ $\boxed{b = -a}$

$4a^2 = 1$

$a = \pm\frac{1}{2}$

$a^2 + 1 \Rightarrow \frac{1}{4} + 1 = \boxed{\frac{5}{4}}$ فاصله از مبداء مختصات ۲

(۸) $y = ax \Rightarrow f'(x) = \frac{1}{\sqrt{x}}(4x^2 + c) + 8x(2\sqrt{x}) = a$

$f'(x) = \frac{4x^2 + c + 16x^2}{\sqrt{x}} = a\sqrt{x}$

$2\sqrt{x}(4x^2 + c) = a x^{\sqrt{x}}$

$20x^2 + 3 = 8x^2 + 9$

$12x^2 = 3 \rightarrow x = \pm\frac{1}{2}$ غ ق ق

$a = \sqrt{2} \times 4 + 4 \times \sqrt{2} = \boxed{8\sqrt{2}}$ شیب خط مماس ✓ (۲)

---

(۹) $y = ax \quad f'(x) = \frac{\frac{1}{2\sqrt{x}}(-2x^2 + x + 1) - (-4x + 1)\sqrt{x}}{(-2x^2 + x + 1)^2} = a$

$-2x^2 + x + 1 - 2x(-4x + 1)$ $(+8x^2 - 2x)$ $\Rightarrow \frac{9x^2 - x + 1}{2\sqrt{x}(-2x^2 + x + 1)^2} = a\sqrt{x}$

(۱,۵)

$ax = \frac{\sqrt{x}}{-2x^2 + x + 1} \Rightarrow \frac{1}{\text{مخرج}} = \frac{9x^2 - x + 1}{2(\text{مخرج})^2} \Rightarrow -4x^2 + 2x + 1 = 9x^2 - x + 1$

$10x^2 = 3x$

$x \to 0$ غ ق ق , $x \to 0{,}3$ ✓

$A \,|\, 0{,}3 \Rightarrow \frac{\sqrt{0{,}3}}{-0{,}18 + 0{,}3 + 1} = \boxed{\frac{\sqrt{0{,}3}}{1{,}12}}$

---

(۱۰) $(f \circ g)' \Rightarrow f'(2) \times g'(\frac{\sqrt{5}}{2}) \Rightarrow 3x^2 \times \frac{-2x}{2\sqrt{x^2 - 1} \times (x^2 - 1)} \Rightarrow 12 \times \frac{-\sqrt{5}}{\frac{1}{4}} = -4\sqrt{5}$ (۱,۵)

$[2^-] = 1 \;/\; g(\frac{\sqrt{5}}{2}) = 2$

جواب

$\frac{-4\sqrt{5}}{48\sqrt{5}} = \boxed{\frac{1}{12}}$

۹

$$y = mx \to \frac{\sqrt{a}}{-2a^2+a+1} = ma \to \frac{1}{-2a^2+a+1} = m\sqrt{a}$$

$$m\sqrt{a}(-2a^2+a+1) = 1 \to -2m(a^{\frac{5}{2}}) + m(a^{\frac{3}{2}}) + m(a)^{\frac{1}{2}} = 1 \xrightarrow{\text{مشتق}}$$

$$-5m(a^{\frac{3}{2}}) + \frac{3}{2}m(a^{\frac{1}{2}}) + \frac{m}{2}(a^{-\frac{1}{2}}) = 0$$

$$\frac{m}{2}(a^{-\frac{1}{2}})(-10a^2+3a+1) = 0 \to a = -\frac{1}{5} \text{ یا } a = \frac{1}{2} \checkmark \ (a>0)$$

$$f(a) = \frac{\frac{\sqrt{2}}{2}}{-2(\frac{1}{4}) + \frac{1}{2} + 1} = \frac{\frac{\sqrt{2}}{2}}{1} = \boxed{\frac{\sqrt{2}}{2}}$$

۱۰

$$g(x) = (x^2-1)^{-\frac{1}{2}} \to g'(x) = -\frac{1}{2}(2x)(x^2-1)^{-\frac{3}{2}}$$

$$g'(\frac{\sqrt{5}}{2}) = -\frac{1}{2}(\sqrt{5})(\frac{5}{4}-1)^{-\frac{3}{2}} \to -\frac{\sqrt{5}}{2}(\cancel{2^{-2(-\frac{3}{2})}})_{8} = -4\sqrt{5}$$

$$g(\frac{\sqrt{5}}{2}^-) = \frac{1}{\sqrt{\frac{5^-}{4}-1}} = \frac{1}{\sqrt{\frac{1}{4}^-}} = \frac{1}{\frac{1}{2}^-} = 2^+$$

$$f'(2^+) = ((2x)^3)' = 24x^2 = 24 \times 4$$

$$fog'(\frac{\sqrt{5}^-}{2}) = -4\sqrt{5} \times 24 \times 4 \xrightarrow{\div -48\sqrt{5}} \frac{\cancel{24}^{2} \times \cancel{4} \times -4\sqrt{5}}{-\cancel{48}\sqrt{5}} = \boxed{8}$$