Date No

آزمایش کتاب تکلیف ۲۴:

1. $\left.\begin{matrix}A(2,a)\\B(0,1)\end{matrix}\right\} m = \frac{a-1}{2-0} = \frac{4}{2} \Rightarrow f'(2) = \frac{4}{2}$

2. $\left.\begin{matrix}B(2,2)\\C(-1,1)\end{matrix}\right\} m = \frac{1}{3}$ $\quad A\left|\begin{matrix}x_0\\\sqrt{ax_0-1}\end{matrix}\right.$ $\rightsquigarrow \frac{x}{3} + \frac{\varepsilon}{3} = \sqrt{ax-1}$

$\hookrightarrow y = \frac{x}{3} + \frac{4}{3}$ $\quad (x+\varepsilon)^2 = 3\sqrt{ax-1}$

$\hookrightarrow x^2 + (\lambda - 9a)x + 2\Delta = 0$

$\lambda - 9a = 10 \rightsquigarrow -9a = 2 \Rightarrow a = \frac{-2}{9}$

$\lambda - 9a = -10 \rightsquigarrow -9a = -18 \Rightarrow a = 2 \rightarrow x^2 - 10x + 2\Delta = 0 \Rightarrow x = \Delta$

$f(\Delta) = \sqrt{2(\Delta)-1} = 3$

3. $y' = \frac{(2x+m)(x+3) - (x^2+mx+1)}{(x+3)^2} \xrightarrow{x=1} \frac{4(2+m) - (m+7)}{16} = \frac{3(m-2)}{16}$

$4y - 3x = 0 \rightsquigarrow m = \frac{-(-3)}{4} = \frac{3}{4}$ $\quad \frac{3(m-2)}{16} = \frac{3}{4} \rightsquigarrow m-2 = 4 \Rightarrow m = 2$

$y = \frac{x^2+2x+1}{x+3} \Rightarrow (1,1)$ $\quad f(1) = 3(1) + n \Rightarrow n = 1$ $\quad m+n = 2+1 = 3$

4. $g'(x) = \frac{3+\sin x - 3\cos x}{(3+\sin x)^2}$ $\quad f(x) = \frac{(3-\sin x)(\sin^2 x + 3\sin x + 9)}{(3-\sin x)(3+\sin x)} = \frac{\sin^2 x + 3\sin x + 9}{\sin x + 3}$

$(3g(x) - f(x))' = \left(\frac{9 - (9 + 3\sin x + \sin^2 x)}{3+\sin x}\right)' = (-\sin x)' = -\cos x$

$x = \frac{5\pi}{3} \rightarrow -\cos\frac{5\pi}{3} = \frac{-1}{2}$

5. $x > 0 \rightarrow g(x) = \frac{1}{2x^5}$ $\quad g(x) > 0 \rightarrow f(x) = \frac{-1}{\sqrt[5]{2x}}$ $\rightsquigarrow \log(x) = \frac{-1}{\sqrt[5]{2(\frac{1}{2x^5})}}$

$\rightarrow \log(x) = -x \Rightarrow (\log)'(x) = -1 \rightsquigarrow (\log)'(\sqrt[5]{3}) = -1$

6. $f(x) = \left(\frac{\sin x - 1}{\sin x + 1}\right)^2 = \left(1 + \frac{-2}{\sin x + 1}\right)^2 = 1 + \underbrace{\frac{-4}{\sin x + 1} + \frac{4}{(\sin x + 1)^2}}_{x g(x)}$

$x g(x) = \frac{-4}{\sin x + 1}\left(\frac{\sin x}{\sin x + 1}\right) \rightsquigarrow$

$\lim_{x \to 0} g(x) = \underbrace{\frac{-4}{\sin x + 1}\left(\frac{\sin x}{\sin x + 1}\right)}_{x} \xrightarrow{\sin x \sim x} \frac{-4}{(\sin x + 1)^2} \Rightarrow \lim_{x \to 0} g(x) = -4$

7. تمرین: $-x^2 - 1 \xrightarrow{\text{مشتق}} y' = -2x$ نقاط برخورد با $A(a, \beta)$ $B(-a, \beta)$

$m_1 = -2(a)$ $m_2 = -2(-a) \rightsquigarrow m_1 m_2 = -1 \rightsquigarrow 4a^2 = 1 \Rightarrow a = \pm\frac{1}{2}$

$\Rightarrow A\left(\frac{1}{2}, \beta\right)$ $B\left(-\frac{1}{2}, \beta\right)$ نقطه از جنس β: $\beta = -\left(\pm\frac{1}{2}\right)^2 - 1 = -\frac{5}{4}$

8. $f'(x) = \frac{1}{\sqrt{x}}(4x^2 + 3) + (8x)(2\sqrt{x}) = \frac{20x^2 + 3}{\sqrt{x}}$ $(y - y_0) = m(x - x_0)$

$\rightarrow (y - (2\sqrt{a})(4a^2 + 3)) = \frac{20a^2 + 3}{\sqrt{a}}(x - a) \xrightarrow{(0,0)} -2\sqrt{a}(4a^2 + 3) = \frac{2a^2 + 3}{\sqrt{a}}(-a)$

$2(4a^2 + 3) = 20a^2 + 3 \Rightarrow a^2 = \frac{1}{4}$ $m = \frac{20a^2 + 3}{\sqrt{a}} = \frac{20\left(\frac{1}{4}\right) + 3}{\sqrt{\frac{1}{2}}} = 8\sqrt{2}$

10. $(fog(x))' = g'(x) f'(g(x))$ $g'(x) = \frac{-1}{2}(x^2 - 1)^{-\frac{3}{2}} \times 2x$

$g'\left(\frac{\sqrt{5}}{2}\right) = \frac{1}{\sqrt{\left(1 - \left(\frac{3}{5}\right)\right)}} = \frac{1}{\sqrt{\frac{1}{5}}} = 2$

$f'(x) = f'(x^2) = ((2x^2))' = 4x^3 = 32 \quad f'(2) = 4 \times 32 = 128$

$(fog(x))' = -4\sqrt{5} \times 32 \times 4 \div 8\sqrt{5}$ برابر $\rightarrow$