نام: دینا احمدی — شماره تکلیف: ۲۴ — مقطع: دوازدهم دختران B

۱۹٫۵ آفرین

①
$(0,1)\,(3,5) \to f'(3)=\frac{\Delta y}{\Delta x}=\frac{4}{3}$ ۲

②
شیب گذرنده از $(2,2),(-1,1)$: $\to m=\frac{2-1}{2+1}=\frac{1}{3} \to y=\frac{1}{3}x+\frac{4}{3}$

$f'(x)=\frac{a}{2\sqrt{ax-1}} \to \frac{a}{2\sqrt{ax-1}}=\frac{1}{3} \to 3a=2\sqrt{ax-1}$ ①

$x_0=t \to \sqrt{at-1}=\frac{t+4}{3}$ ② $\xrightarrow{①,②} 3a=2\left(\frac{t+4}{3}\right) \to a=\frac{2(t+4)}{9} \xrightarrow{*} a=\frac{2(5+4)}{9}=2$

$\downarrow$

$\frac{2t(t+4)}{9}-1=\frac{(t+4)^2}{9} \to 2t(t+4)-9=(t+4)^2 \to t^2=25 \to \begin{cases} t=5 \ ✓ \ * \\ t=-5 \ ✗ \end{cases}$ زیرا رادیکال منفی

$f(5)=\sqrt{2x-1} \to f(5)=3$ ۲

③
$y=\frac{3}{4}x+\frac{n}{4}$ ① $\to f'(x)=\frac{3}{4}$

$f'(x)=\frac{(2x+m)(x+3)-(x^2+mx+1)}{(x+3)^2} = \frac{x^2+6x+(3m-1)}{x^2+6x+9}=\frac{3}{4}$ ۲

$\xrightarrow{x=1} \frac{7+(3m-1)}{16}=\frac{3}{4} \to 3m-1=5 \to m=2 \to m+n=3$

$g(1)=1 \to (1,1)$ مس $\xrightarrow{①} 1=\frac{3}{4}+\frac{n}{4} \to n=1$

④
$g'(x)=\frac{-2\cos x}{(3+\sin x)^2} \to g'\left(\frac{5\pi}{3}\right)=\frac{-24-8\sqrt{3}}{121}$

$f(x)=\frac{(3-\sin x)(9+3\sin x+\sin^2 x)}{(3-\sin x)(3+\sin x)}=\sin x+\frac{9}{3+\sin x} \to f'(x)=\cos x\left(1-\frac{9}{(3+\sin x)^2}\right)$ ۲

$\to f'\left(\frac{5\pi}{3}\right)=\frac{-35-48\sqrt{3}}{242} \Rightarrow 2f'\left(\frac{5\pi}{3}\right)+g'\left(\frac{5\pi}{3}\right)=\frac{-121}{242}=\frac{-1}{2}$

⑤
$g(x)=\frac{1}{2x^5} \to g(\sqrt[5]{3})=\frac{1}{6}$ $\quad g'(x)=\frac{-5}{2x^6} \to g'(\sqrt[5]{3})=\frac{-5}{6\sqrt[5]{3}}$

$f(x)=-(2x)^{-\frac{1}{5}} \to f'(x)=\frac{2}{5}(2x)^{-\frac{6}{5}} \to f'\left(\frac{1}{6}\right)=\frac{6}{5}\sqrt[5]{3}$

$g'(\sqrt[5]{3})\times f'\left(\frac{1}{6}\right)=\frac{-5}{6\sqrt[5]{3}}\times\frac{6}{5}\sqrt[5]{3}=-1$ ۲

⑥
$x\to 0 \Rightarrow f(x)=\left(\frac{-1+0}{1+0}\right)^2=1 \to f(x)=xg(x)+1 \xrightarrow[f(0)=1]{} x.g(x)=0$ ۲

$f'(x)=g(x)+xg'(x) \xrightarrow{*} g(0)=-4$ اگر پیوسته باشد $\lim_{x\to 0} g(x)=-4$

$f'(x)=2\left(\frac{-1+\sin x}{1+\sin x}\right)\left(\frac{\cos x(1+\sin x)-(\sin x-1)(\cos x)}{(1+\sin x)^2}\right) \to f'(0)=-4$ *

⑦

$\to -x^2-1$

$f(x)=-x^2-1 \to f'(x)=-2x$

$f'(x).f'(-x)=-1 \to (-2x)(2x)=-1 \to x=\pm\frac{1}{2}$ ۲

$f(x)=f(-x) \to -x^2-1=\frac{-1}{4}-1=\frac{-5}{4} \to d \Rightarrow y=\frac{-5}{4} \to$ فاصله d تا مبدا $\frac{5}{4}$ است.

(8)

$$f'(x)=\frac{2}{2\sqrt{x}}(4x^2+3)+2\sqrt{x}\,(8x)=\frac{4x^2+3}{\sqrt{x}}+16x\sqrt{x}=\frac{20x^2+3}{\sqrt{x}}\xrightarrow{\text{مس}}\frac{20x^2+3}{\sqrt{x}}=\frac{2(4x^2+3)}{\sqrt{x}}$$

$$\rightarrow 12x^2=3\rightarrow x^2=\frac{1}{4}\ \begin{cases} x=\frac{-1}{2}\ \times \quad D_f: x>0 \\ x=\frac{1}{2}\ \checkmark \end{cases}\ \longrightarrow m=\frac{20\left(\frac{1}{2}\right)^2+3}{\sqrt{\frac{1}{2}}}=\boxed{8\sqrt{2}}$$

2

(9)

$A(a,f(a))$, $y=mx$, $m=f'(a)$ $\quad m=\frac{f(a)-0}{a-0}=\frac{f(a)}{a}=f'(a)$

$$\frac{\frac{1}{2\sqrt{a}}(-2a^2+a+1)-\sqrt{a}(-4a+1)}{(-2a^2+a+1)^2}=\frac{1}{\sqrt{a}(-2a^2+a+1)}\longrightarrow 10a^2-3a-1=0\ \begin{cases} a=\frac{-1}{5}\ \times\ \text{در } D_f \text{ نیست} \\ a=\boxed{\frac{1}{2}} \end{cases}$$

$$\xrightarrow[f(x)]{\text{جایگذاری در}} A(a,f(a))\longrightarrow A\left(\frac{1}{2},\frac{\sqrt{2}}{2}\right)\longrightarrow A \text{ عرض}=\frac{\sqrt{2}}{2}$$

2

(10)

$$g\left(\frac{\sqrt{5}}{2}\right)=\frac{1}{\sqrt{\frac{5}{4}-1}}=2 \qquad g'(x)=\frac{2x}{2\sqrt{x^2-1}}=\frac{x}{\sqrt{x^2-1}}\longrightarrow g'\left(\frac{\sqrt{5}}{2}\right)=\frac{\frac{\sqrt{5}}{2}}{\frac{1}{2}}=\sqrt{5}$$

1/5

$$f(x)=(\underbrace{x[x]}_{A})^3\longrightarrow f'(x)=3A^2A'=3(x[x])^2([x])\longrightarrow f'_-(2)=3(2\times1)^2\times1=12$$

$$f\circ g'\left(\frac{\sqrt{5}}{2}\right)=g'\left(\frac{\sqrt{5}}{2}\right)\cdot f'\left(g\left(\frac{\sqrt{5}}{2}\right)\right)=12\times\sqrt{5}=12\sqrt{5}\longrightarrow\frac{12\sqrt{5}}{-48\sqrt{5}}=\frac{-1}{4}\ \text{برابر}$$

۱۰

$$g(x) = (x^{۲}-۱)^{-\frac{۱}{۲}} \longrightarrow g'(x) = -\frac{۱}{۲}(۲x)(x^{۲}-۱)^{-\frac{۳}{۲}}$$

$$g'\left(\frac{\sqrt{۵}}{۲}\right) = -\frac{۱}{۲}(\sqrt{۵})\left(\frac{۵}{۴}-۱\right)^{-\frac{۳}{۲}} \longrightarrow -\frac{\sqrt{۵}}{۲}\left(۴^{-۲(-\frac{۳}{۲})}\right) = -۴\sqrt{۵}$$

$$۴^{-۲(-\frac{۳}{۲})} = ۸$$

$$g\left(\frac{\sqrt{۵}}{۲}^{-}\right) = \frac{۱}{\sqrt{\frac{۵^{-}}{۴}-۱}} = \frac{۱}{\sqrt{\frac{۱}{۴}^{-}}} = \frac{۱}{\frac{۱}{۲}^{-}} = ۲^{+}$$

$$f'(۲^{+}) = ((۲x)^{۳})' = ۲۴x^{۲} = ۲۴\times۴$$

$$fog'\left(\frac{\sqrt{۵}^{-}}{۲}\right) = -۴\sqrt{۵}\times ۲۴\times ۴ \xrightarrow{\div -۴۸\sqrt{۵}} \frac{۲۴\times۴\times -۴\sqrt{۵}}{-۴۸\sqrt{۵}} = \boxed{۸}$$

$$\frac{۲۴}{-۴۸} \to ۲$$