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① $(0,1)\ (3,5) \to f'(3) = \frac{\Delta y}{\Delta x} = \frac{4}{3}$

② شیب گذرنده از $(2,2),(-1,1)$ $\to m = \frac{2-1}{2+1} = \frac{1}{3} \to y = \frac{1}{3}x + \frac{4}{3}$ $\quad f'(x) = \frac{a}{2\sqrt{ax-1}} \to \frac{a}{2\sqrt{ax-1}} = \frac{1}{3} \to 3a = 2\sqrt{ax-1}$ ①

$\xrightarrow{x_0 = t} \sqrt{at-1} = \frac{t+4}{3}$ ② $\xrightarrow{①,②} 3a = 2\left(\frac{t+4}{3}\right) \to a = \frac{2(t+4)}{9} \xrightarrow{*} a = \frac{2(5+4)}{9} = 2$

$\downarrow$

$\frac{2t(t+4)}{9} - 1 = \frac{(t+4)^2}{9} \to 2t(t+4) - 9 = (t+4)^2 - t^2 = 25 \to \begin{cases} t = 5 \ \checkmark \ * \\ t = -5 \ \times \text{ زیر رادیکال منفی} \end{cases}$

$f(5) = \sqrt{2x-1} \to f(5) = \boxed{3}$

③ $y = \frac{3}{4}x + \frac{n}{4}$ ① $\to f'(x) = \frac{3}{4}$

$f'(x) = \frac{(2x+m)(x+3) - (x^2+mx+1)}{(x+3)^2} = \frac{x^2+6x+(3m-1)}{x^2+6x+9} = \frac{3}{4}$

$\xrightarrow{x=1} \frac{7+(3m-1)}{16} = \frac{3}{4} \to 3m-1 = 5 \to m = 2 \quad \to m+n = \boxed{3}$

$y(1) = 1 \to \underset{\text{مس}}{(1,1)} \xrightarrow{①} 1 = \frac{3}{4} + \frac{n}{4} \to n = 1$

④ $g'(x) = \frac{-2\cos x}{(3+\sin x)^2} \to g'\left(\frac{5\pi}{3}\right) = \frac{-26-8\sqrt{3}}{121}$

$f(x) = \frac{(3-\sin x)(9+3\sin x+\sin^2 x)}{(3-\sin x)(3+\sin x)} = \sin x + \frac{9}{3+\sin x} \to f'(x) = \cos x\left(1 - \frac{9}{(3+\sin x)^2}\right)$

$\to f'\left(\frac{5\pi}{3}\right) = \frac{-35-48\sqrt{3}}{242} \Rightarrow 2f'\left(\frac{5\pi}{3}\right) + g'\left(\frac{5\pi}{3}\right) = \frac{-121}{242} = \frac{-1}{2}$

⑤ $g(x) = \frac{1}{2x^5} \to g(\sqrt[5]{3}) = \frac{1}{6} \quad g'(x) = \frac{-5}{2x^6} \to g'(\sqrt[5]{3}) = \frac{-5}{6\sqrt[5]{3}}$

$f(x) = -(2x)^{-\frac{2}{5}} \to f'(x) = \frac{2}{5}(2x)^{-\frac{6}{5}} \to f'\left(\frac{1}{6}\right) = \frac{6}{5}\sqrt[5]{3}$

$g'(\sqrt[5]{3}) \times f'\left(\frac{1}{6}\right) = \frac{-5}{6\sqrt[5]{3}} \times \frac{6}{5}\sqrt[5]{3} = \boxed{-1}$

⑥ $x \to 0 \Rightarrow f(x) = \left(\frac{-1+0}{1+0}\right)^2 = 1 \to f(x) = xg(x) + 1 \xrightarrow[f(x)=1]{} x \cdot g(x) = 0$

$f'(x) = g(x) + xg'(x) \xrightarrow{*} g(0) = \boxed{-4}$ اگر پیوسته $\lim_{x\to 0} g(x) = \boxed{-4}$

$f'(x) = 2\left(\frac{-1+\sin x}{1+\sin x}\right)\left(\frac{\cos x(1+\sin x) - (\sin x - 1)(\cos x)}{(1+\sin x)^2}\right) \to f'(0) = -4 \ *$

⑦  $\to -x^2-1$

$f(x) = -x^2-1 \to f'(x) = -2x$

$f'(x) \cdot f'(-x) = -1 \to (-2x)(2x) = -1 \to x = \pm\frac{1}{2}$

$f(x) = f(-x) \to -x^2-1 = \frac{-1}{4} - 1 = \frac{-5}{4} \to d \Rightarrow y = \frac{-5}{4} \to$ پس فاصله d تا مبدا $\frac{5}{4}$ است.

(8)

$$f'(x)=\frac{2}{2\sqrt{x}}(4x^2+3)+2\sqrt{x}\,(8x)=\frac{4x^2+3}{\sqrt{x}}+16x\sqrt{x}=\frac{20x^2+3}{\sqrt{x}}\xrightarrow{\text{مس}}\frac{20x^2+3}{\sqrt{x}}=\frac{2(4x^2+3)}{\sqrt{x}}$$

$$\to 12x^2=3\to x^2=\frac{1}{4}\ \begin{cases}x=\frac{-1}{2}\ \times\\ x=\frac{1}{2}\ \checkmark\end{cases}\quad D_f: x>0\ \longrightarrow m=\frac{20\left(\frac{1}{2}\right)^2+3}{\sqrt{\frac{1}{2}}}=\boxed{8\sqrt{2}}$$

(9)

$$A(a,f(a)),\ y=mx,\ m=f'(a)\qquad m=\frac{f(a)-0}{a-0}=\frac{f(a)}{a}=f'(a)$$

$$\frac{\frac{1}{2\sqrt{a}}(-2a^2+a+1)-\sqrt{a}\,(-4a+1)}{(-2a^2+a+1)^2}=\frac{1}{\sqrt{a}\,(-2a^2+a+1)}\longrightarrow 10a^2-3a-1=0\ \begin{cases}a=\frac{-1}{5}\ \times\ \text{در } D_f \text{ نیست}\\ a=\boxed{\frac{1}{2}}\end{cases}$$

$$\xrightarrow[f(x)]{\text{جایگذاری در}} A(a,f(a))\longrightarrow A\left(\frac{1}{2},\frac{\sqrt{2}}{2}\right)\longrightarrow A\ \text{عرض}=\frac{\sqrt{2}}{2}$$

(10)

$$g\left(\frac{\sqrt{5}}{2}\right)=\frac{1}{\sqrt{\frac{5}{4}-1}}=2\qquad g'(x)=\frac{2x}{2\sqrt{x^2-1}}=\frac{x}{\sqrt{x^2-1}}\longrightarrow g'\left(\frac{\sqrt{5}}{2}\right)=\frac{\frac{\sqrt{5}}{2}}{\frac{1}{2}}=\sqrt{5}$$

$$f(x)=(\underbrace{x[x]}_{A})^3\longrightarrow f'(x)=3A^2A'=3(x[x])^2([x])\longrightarrow f'_-(2)=3(2\times1)^2\times1=12$$

$$fog\left(\frac{\sqrt{5}}{2}\right)=g'\left(\frac{\sqrt{5}}{2}\right).f'\left(g\left(\frac{\sqrt{5}}{2}\right)\right)=12\times\sqrt{5}=12\sqrt{5}\longrightarrow\frac{12\sqrt{5}}{-48\sqrt{5}}=\frac{-1}{4}\quad \text{برابر}$$